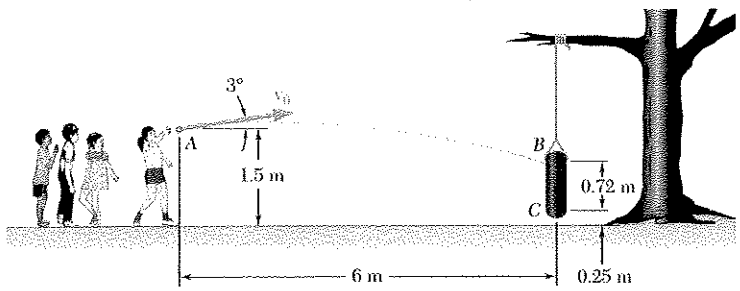


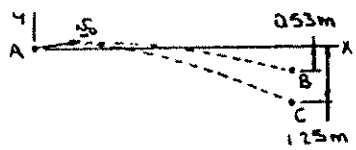
PROBLEM 11.107

A group of children are throwing balls through a 0.72-m-inner-diameter tire hanging from a tree. A child throws a ball with an initial velocity $\mathbf{v}_0$ at an angle of 3° with the horizontal. Determine the range of values of v_0 for which the ball will go through the tire.

SOLUTION

First note

$$(v_x)_0 = v_0 \cos 3°$$
$$(v_y)_0 = v_0 \sin 3°$$



Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

When $x = 6$ m: $\quad 6 = (v_0 \cos 3°)t \quad$ or $\quad t_6 = \dfrac{6}{v_0 \cos 3°}$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2 \qquad (g = 9.81 \text{ m/s}^2)$$

When the ball reaches the tire, $t = t_6$

$$y_{B,C} = (v_0 \sin 3°)\left(\frac{6}{v_0 \cos 3°}\right) - \frac{1}{2} g \left(\frac{6}{v_0 \cos 3°}\right)^2$$

or $$v_0^2 = \frac{18(9.81)}{\cos^2 3°(6 \tan 3° - y_{B,C})}$$

or $$v_0^2 = \frac{177.065}{0.314447 - y_{B,C}}$$

At B, $y = -0.53$ m: $$v_0^2 = \frac{177.065}{0.314447 - (-0.53)}$$

or $$(v_0)_B = 14.48 \text{ m/s}$$

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PROBLEM 11.107 (Continued)

At C, $y = -1.25$ m: $$v_0^2 = \frac{177.06 \text{ s}}{0.314447 - (-1.25)}$$

or $$(v_0)_C = 10.64 \text{ m/s}$$

$$10.64 \text{ m/s} \leq v_0 \leq 14.48 \text{ m/s} \blacktriangleleft$$

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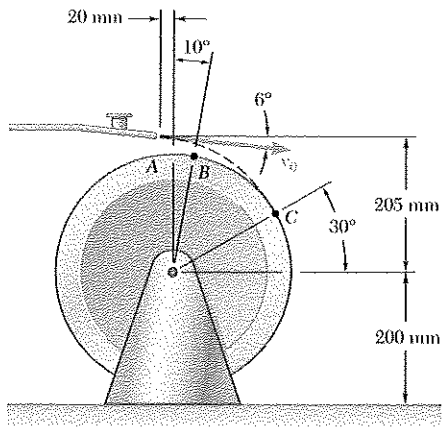


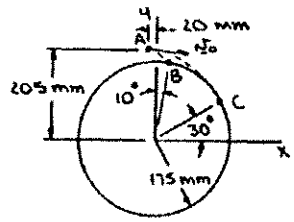
PROBLEM 11.108

The nozzle at A discharges cooling water with an initial velocity $\mathbf{v}_0$ at an angle of 6° with the horizontal onto a grinding wheel 350 mm in diameter. Determine the range of values of the initial velocity for which the water will land on the grinding wheel between Points B and C.

SOLUTION

First note

$$(v_x)_0 = v_0 \cos 6°$$
$$(v_y)_0 = -v_0 \sin 6°$$



Horizontal motion. (Uniform)

$$x = x_0 + (v_x)_0 t$$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B: $x = (0.175 \text{ m}) \sin 10°$

$$y = (0.175 \text{ m}) \cos 10°$$

x: $0.175 \sin 10° = -0.020 + (v_0 \cos 6°) t$

or $$t_B = \frac{0.050388}{v_0 \cos 6°}$$

y: $$0.175 \cos 10° = 0.205 + (-v_0 \sin 6°) t_B - \frac{1}{2} g t_B^2$$

Substituting for t_B

$$-0.032659 = (-v_0 \sin 6°)\left(\frac{0.050388}{v_0 \cos 6°}\right) - \frac{1}{2}(9.81)\left(\frac{0.050388}{v_0 \cos 6°}\right)^2$$

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PROBLEM 11.108 (Continued)

or $$v_0^2 = \frac{\frac{1}{2}(9.81)(0.050388)^2}{\cos^2 6°(0.032659 - 0.050388 \tan 6°)}$$

or $$(v_0)_B = 0.678 \text{ m/s}$$

At Point C: $$x = (0.175 \text{ m}) \cos 30°$$
$$y = (0.175 \text{ m}) \sin 30°$$

x: $$0.175 \cos 30° = -0.020 + (v_0 \cos 6°)t$$

or $$t_C = \frac{0.171554}{v_0 \cos 6°}$$

y: $$0.175 \sin 30° = 0.205 + (-v_0 \sin 6°)t_C - \frac{1}{2} g t_C^2$$

Substituting for t_C

$$-0.117500 = (-v_0 \sin 6°)\left(\frac{0.171554}{v_0 \cos 6°}\right) - \frac{1}{2}(9.81)\left(\frac{0.171554}{v_0 \cos 6°}\right)^2$$

or $$v_0^2 = \frac{\frac{1}{2}(9.81)(0.171554)^2}{\cos^2 6°(0.117500 - 0.171554 \tan 6°)}$$

or $$(v_0)_C = 1.211 \text{ m/s}$$

$$0.678 \text{ m/s} \le v_0 \le 1.211 \text{ m/s} \blacktriangleleft$$

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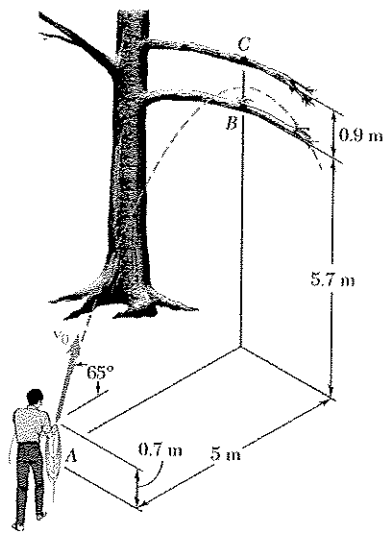


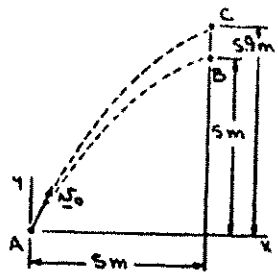
PROBLEM 11.109

While holding one of its ends, a worker lobs a coil of rope over the lowest limb of a tree. If he throws the rope with an initial velocity $\mathbf{v}_0$ at an angle of 65° with the horizontal, determine the range of values of v_0 for which the rope will go over only the lowest limb.

SOLUTION

First note

$$(v_x)_0 = v_0 \cos 65°$$
$$(v_y)_0 = v_0 \sin 65°$$



Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At either B or C, $x = 5$ m

$$s = (v_0 \cos 65°) t_{B,C}$$

or

$$t_{B,C} = \frac{5}{(v_0 \cos 65°)}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2 \qquad (g = 9.81 \text{ m/s}^2)$$

At the tree limbs, $t = t_{B,C}$

$$y_{B,C} = (v_0 \sin 65°)\left(\frac{5}{v_0 \cos 65°}\right) - \frac{1}{2} g \left(\frac{5}{v_0 \cos 65°}\right)^2$$

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PROBLEM 11.109 (Continued)

or $$v_0^2 = \frac{\frac{1}{2}(9.81)(25)}{\cos^2 65°(5 \tan 65° - y_{B,C})}$$

$$= \frac{686.566}{5 \tan 65° - y_{B,C}}$$

At Point B: $$v_0^2 = \frac{686.566}{5 \tan 65° - 5} \quad \text{or} \quad (v_0)_B = 10.95 \text{ m/s}$$

At Point C: $$v_0^2 = \frac{686.566}{5 \tan 65° - 5.9} \quad \text{or} \quad (v_0)_C = 11.93 \text{ m/s}$$

$$10.95 \text{ m/s} \le v_0 \le 11.93 \text{ m/s} \blacktriangleleft$$

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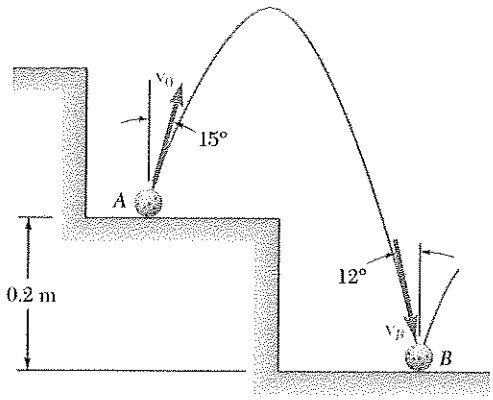


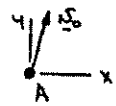
PROBLEM 11.110

A ball is dropped onto a step at Point A and rebounds with a velocity $\mathbf{v}_0$ at an angle of 15° with the vertical. Determine the value of v_0, knowing that just before the ball bounces at Point B, its velocity $\mathbf{v}_B$ forms an angle of 12° with the vertical.

SOLUTION

First note

$$(v_x)_0 = v_0 \sin 15°$$
$$(v_y)_0 = v_0 \cos 15°$$



Horizontal motion. (Uniform)

$$v_x = (v_x)_0 = v_0 \sin 15°$$

Vertical motion. (Uniformly accelerated motion)

$$v_y = (v_y)_0 - gt \qquad y = 0 + (v_y)_0 t - \frac{1}{2} g t^2$$
$$= v_0 \cos 15° - gt \qquad = (v_0 \cos 15°) t - \frac{1}{2} g t^2$$

At Point B, $v_y < 0$

Then

$$\tan 12° = \frac{(v_x)_B}{|(v_y)_B|} = \frac{v_0 \sin 15°}{g t_B - v_0 \cos 15°}$$

or

$$t_B = \frac{v_0}{g}\left(\frac{\sin 15°}{\tan 12°} + \cos 15°\right) \qquad g = 9.81 \text{ m/s}^2$$
$$= 0.22259\, v_0$$

Noting that

$$y_B = -0.2 \text{ m},$$

We have

$$-0.2 = (v_0 \cos 15°)(0.22259\, v_0) - \frac{1}{2}(9.81)(0.22259\, v_0)^2$$

or

$$v_0 = 2.67 \text{ m/s} \blacktriangleleft$$

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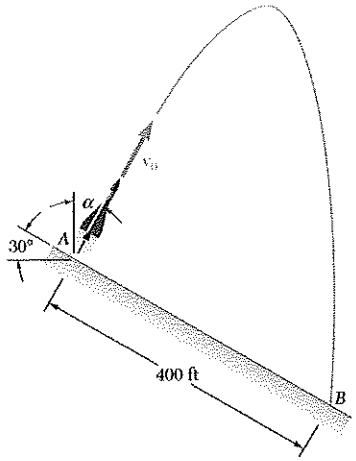

PROBLEM 11.111

A model rocket is launched from Point A with an initial velocity $\mathbf{v}_0$ of 250 ft/s. If the rocket's descent parachute does not deploy and the rocket lands 400 ft from A, determine (a) the angle α that $\mathbf{v}_0$ forms with the vertical, (b) the maximum height above Point A reached by the rocket, and (c) the duration of the flight.

SOLUTION

Set the origin at Point A. $\quad x_0 = 0, \quad y_0 = 0$

Horizontal motion: $\quad x = v_0 t \sin\alpha \qquad \sin\alpha = \dfrac{x}{v_0 t}$ (1)

Vertical motion: $\quad y = v_0 t \cos\alpha - \dfrac{1}{2} g t^2$

$$\cos\alpha = \frac{1}{v_0 t}\left(y + \frac{1}{2} g t^2\right) \tag{2}$$

$$\sin^2\alpha + \cos^2\alpha = \frac{1}{(v_0 t)^2}\left[x^2 + \left(y + \frac{1}{2} g t^2\right)^2\right] = 1$$

$$x^2 + y^2 + g y t^2 + \frac{1}{4} g^2 t^4 = v_0^2 t^2$$

$$\frac{1}{4} g^2 t^4 - \left(v_0^2 - g y\right) t^2 + (x^2 + y^2) = 0 \tag{3}$$

At Point B, $\quad \sqrt{x^2 + y^2} = 400 \text{ ft}, \quad x = 400 \cos 30° \text{ ft}$

$$y = -400 \sin 30° = -200 \text{ ft}$$

$$\frac{1}{4}(32.2)^2 t^4 - [250^2 - (32.2)(-200)] t^2 + 400^2 = 0$$

$$259.21 t^4 - 68940 t^2 + 160000 = 0$$

$$t^2 = 263.62 \text{ s}^2 \quad \text{and} \quad 2.34147 \text{ s}^2$$

$$t = 16.2364 \text{ s} \quad \text{and} \quad 1.53019 \text{ s}$$

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PROBLEM 11.111 (Continued)

Restrictions on α: $0 \le \alpha \le 120°$

$$\tan\alpha = \frac{x}{y + \frac{1}{2}gt^2} = \frac{400\cos 30°}{-200 + (16.1)(16.2364)^2} = 0.085654$$

$$\alpha = 4.8957°$$

and $$\frac{400\cos 30°}{-200 + (16.1)(1.53019)^2} = -2.13435$$

$$\alpha = 115.105°$$

Use $\alpha = 4.8957°$ corresponding to the steeper possible trajectory.

(*a*) Angle α. $\alpha = 4.90°$ ◀

(*b*) Maximum height. $v_y = 0$ at $y = y_{\max}$

$$v_y = v_0\cos\alpha - gt = 0$$

$$t = \frac{v_0\cos\alpha}{g}$$

$$y_{\max} = v_0 t\cos\alpha - \frac{1}{2}gt = \frac{v_0^2\cos^2\alpha}{2g}$$

$$= \frac{(250)^2\cos^2 4.8957°}{(2)(32.2)}$$

$y_{\max} = 963$ ft ◀

(*c*) Duration of the flight. (time to reach *B*)

$t = 16.24$ s ◀

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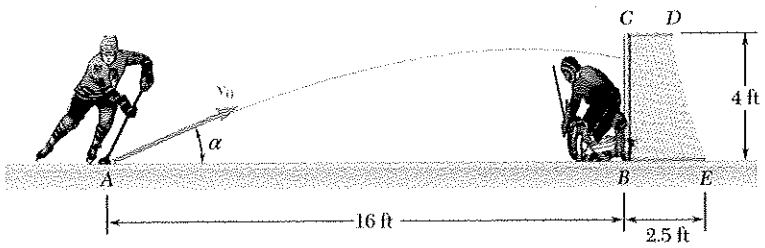

PROBLEM 11.112

The initial velocity $\mathbf{v}_0$ of a hockey puck is 105 mi/h. Determine (*a*) the largest value (less than 45°) of the angle α for which the puck will enter the net, (*b*) the corresponding time required for the puck to reach the net.

SOLUTION

First note $v_0 = 105 \text{ mi/h} = 154 \text{ ft/s}$

and $(v_x)_0 = v_0 \cos\alpha = (154 \text{ ft/s}) \cos\alpha$

$(v_y)_0 = v_0 \sin\alpha = (154 \text{ ft/s}) \sin\alpha$

(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (154 \cos\alpha)t$$

At the front of the net, $x = 16$ ft

Then $16 = (154 \cos\alpha)t$

or $$t_{\text{enter}} = \frac{8}{77 \cos\alpha}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2$$

$$= (154 \sin\alpha)t - \frac{1}{2} g t^2 \quad (g = 32.2 \text{ ft/s}^2)$$

At the front of the net,

$$y_{\text{front}} = (154 \sin\alpha) t_{\text{enter}} - \frac{1}{2} g t_{\text{enter}}^2$$

$$= (154 \sin\alpha)\left(\frac{8}{77 \cos\alpha}\right) - \frac{1}{2} g \left(\frac{8}{77 \cos\alpha}\right)^2$$

$$= 16 \tan\alpha - \frac{32g}{5929 \cos^2\alpha}$$

Now $$\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$$

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PROBLEM 11.112 (Continued)

Then $$y_{\text{front}} = 16 \tan\alpha - \frac{32g}{5929}(1+\tan^2\alpha)$$

or $$\tan^2\alpha - \frac{5929}{2g}\tan\alpha + \left(1+\frac{5929}{32g}y_{\text{front}}\right) = 0$$

Then $$\tan\alpha = \frac{\frac{5929}{2g} \pm \left[\left(-\frac{5929}{2g}\right)^2 - 4\left(1+\frac{5929}{32g}y_{\text{front}}\right)\right]^{1/2}}{2}$$

or $$\tan\alpha = \frac{5929}{4\times 32.2} \pm \left[\left(-\frac{5929}{4\times 32.2}\right)^2 - \left(1+\frac{5929}{32\times 32.2}y_{\text{front}}\right)\right]^{1/2}$$

or $$\tan\alpha = 46.0326 \pm [(46.0326)^2 - (1+5.7541\, y_{\text{front}})]^{1/2}$$

Now $0 \le y_{\text{front}} \le 4$ ft so that the positive root will yield values of $\alpha > 45°$ for all values of y_{front}. When the negative root is selected, α increases as y_{front} is increased. Therefore, for $\alpha_{\max}$, set

$$y_{\text{front}} = y_C = 4 \text{ ft}$$

Then $$\tan\alpha = 46.0326 - [(46.0326)^2 - (1+5.7541+4)]^{1/2}$$

or $$\alpha_{\max} = 14.6604°$$ $$\alpha_{\max} = 14.66° \blacktriangleleft$$

(*b*) We had found $$t_{\text{enter}} = \frac{8}{77\cos\alpha}$$

$$= \frac{8}{77\cos 14.6604°}$$

or $$t_{\text{enter}} = 0.1074 \text{ s} \blacktriangleleft$$

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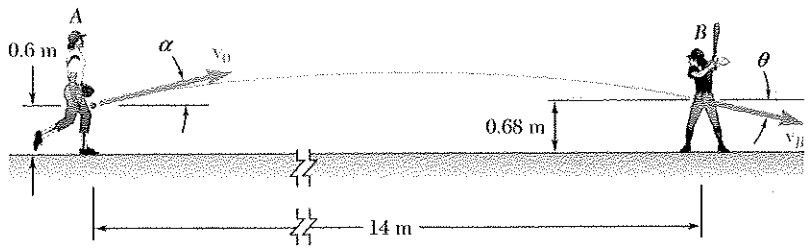
PROBLEM 11.113

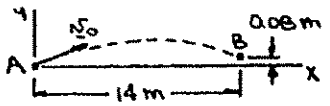


The pitcher in a softball game throws a ball with an initial velocity $\mathbf{v}_0$ of 72 km/h at an angle α with the horizontal. If the height of the ball at Point B is 0.68 m, determine (a) the angle α, (b) the angle θ that the velocity of the ball at Point B forms with the horizontal.

SOLUTION

First note

$$v_0 = 72 \text{ km/h} = 20 \text{ m/s}$$



and

$$(v_x)_0 = v_0 \cos\alpha = (20 \text{ m/s}) \cos\alpha$$
$$(v_y)_0 = v_0 \sin\alpha = (20 \text{ m/s}) \sin\alpha$$

(a) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (20 \cos\alpha)t$$

At Point B:

$$14 = (20 \cos\alpha)t \quad \text{or} \quad t_B = \frac{7}{10 \cos\alpha}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 = (20 \sin\alpha)t - \frac{1}{2}gt^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B:

$$0.08 = (20 \sin\alpha)t_B - \frac{1}{2}gt_B^2$$

Substituting for t_B

$$0.08 = (20 \sin\alpha)\left(\frac{7}{10 \cos\alpha}\right) - \frac{1}{2}g\left(\frac{7}{10 \cos\alpha}\right)^2$$

or

$$8 = 1400 \tan\alpha - \frac{1}{2}g\frac{49}{\cos^2\alpha}$$

Now

$$\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$$

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PROBLEM 11.113 (Continued)

Then $$8 = 1400 \tan\alpha - 24.5g(1 + \tan^2\alpha)$$

or $$240.345 \tan^2\alpha - 1400 \tan\alpha + 248.345 = 0$$

Solving $$\alpha = 10.3786^\circ \quad \text{and} \quad \alpha = 79.949^\circ$$

Rejecting the second root because it is not physically reasonable, we have

$$\alpha = 10.38^\circ \blacktriangleleft$$

(*b*) We have $$v_x = (v_x)_0 = 20 \cos\alpha$$

and $$v_y = (v_y)_0 - gt = 20 \sin\alpha - gt$$

At Point *B*: $$(v_y)_B = 20 \sin\alpha - gt_B$$

$$= 20 \sin\alpha - \frac{7g}{10 \cos\alpha}$$

Noting that at Point *B*, $v_y < 0$, we have

$$\tan\theta = \frac{|(v_y)_B|}{v_x}$$

$$= \frac{\frac{7g}{10\cos\alpha} - 20 \sin\alpha}{20 \cos\alpha}$$

$$= \frac{\frac{7}{200}\frac{9.81}{\cos 10.3786^\circ} - \sin 10.3786^\circ}{\cos 10.3786^\circ}$$

or $$\theta = 9.74^\circ \blacktriangleleft$$

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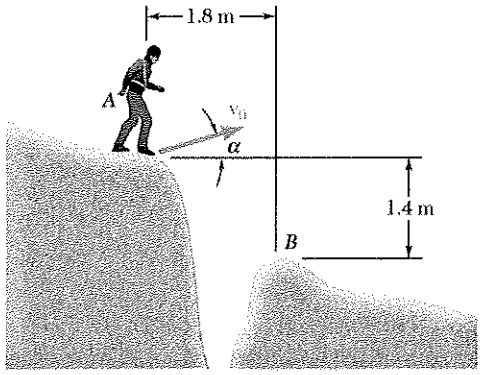


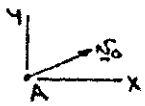
PROBLEM 11.114*

A mountain climber plans to jump from A to B over a crevasse. Determine the smallest value of the climber's initial velocity $\mathbf{v}_0$ and the corresponding value of angle α so that he lands at B.

SOLUTION

First note

$$(v_x)_0 = v_0 \cos\alpha$$
$$(v_y)_0 = v_0 \sin\alpha$$



Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (v_0 \cos\alpha)t$$

At Point B:

$$1.8 = (v_0 \cos\alpha)t$$

or

$$t_B = \frac{1.8}{v_0 \cos\alpha}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2$$
$$= (v_0 \sin\alpha)t - \frac{1}{2}gt^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B:

$$-1.4 = (v_0 \sin\alpha)t_B - \frac{1}{2}gt_B^2$$

Substituting for t_B

$$-1.4 = (v_0 \sin\alpha)\left(\frac{1.8}{v_0 \cos\alpha}\right) - \frac{1}{2}g\left(\frac{1.8}{v_0 \cos\alpha}\right)^2$$

or

$$v_0^2 = \frac{1.62g}{\cos^2\alpha(1.8 \tan\alpha + 1.4)}$$
$$= \frac{1.62g}{0.9 \sin 2\alpha + 1.4 \cos^2\alpha}$$

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PROBLEM 11.114* (Continued)

Now minimize v_0^2 with respect to α.

We have
$$\frac{dv_0^2}{d\alpha} = 1.62g\frac{-(1.8\cos 2\alpha - 2.8\cos\alpha\sin\alpha)}{(0.9\sin 2\alpha + 1.4\cos^2\alpha)^2} = 0$$

or
$$1.8\cos 2\alpha - 1.4\sin 2\alpha = 0$$

or
$$\tan 2\alpha = \frac{18}{14}$$

or
$$\alpha = 26.0625^\circ \quad \text{and} \quad \alpha = 206.06^\circ$$

Rejecting the second value because it is not physically possible, we have

$$\alpha = 26.1^\circ \blacktriangleleft$$

Finally,
$$v_0^2 = \frac{1.62\times 9.81}{\cos^2 26.0625^\circ(1.8\tan 26.0625^\circ + 1.4)}$$

or
$$(v_0)_{\min} = 2.94 \text{ m/s} \blacktriangleleft$$

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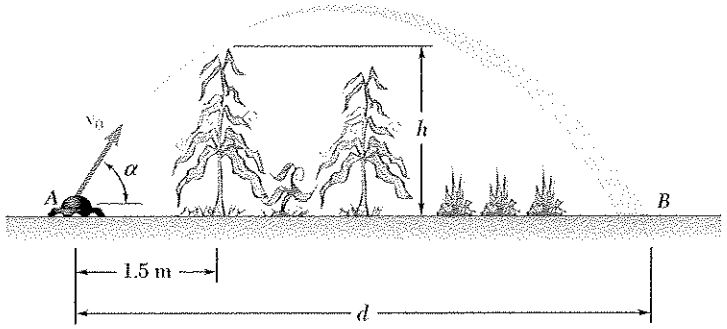

PROBLEM 11.115

An oscillating garden sprinkler which discharges water with an initial velocity $\mathbf{v}_0$ of 8 m/s is used to water a vegetable garden. Determine the distance d to the farthest Point B that will be watered and the corresponding angle α when (*a*) the vegetables are just beginning to grow, (*b*) the height h of the corn is 1.8 m.

SOLUTION

First note
$$(v_x)_0 = v_0 \cos\alpha = (8 \text{ m/s}) \cos\alpha$$
$$(v_y)_0 = v_0 \sin\alpha = (8 \text{ m/s}) \sin\alpha$$

Horizontal motion. (Uniform)
$$x = 0 + (v_x)_0 t = (8 \cos\alpha)t$$

At Point B: $x = d$: $d = (8 \cos\alpha)t$

or
$$t_B = \frac{d}{8 \cos\alpha}$$

Vertical motion. (Uniformly accelerated motion)
$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2$$
$$= (8 \sin\alpha)t - \frac{1}{2} g t^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B:
$$0 = (8 \sin\alpha)t_B - \frac{1}{2} g t_B^2$$

Simplifying and substituting for t_B
$$0 = 8 \sin\alpha - \frac{1}{2} g \left(\frac{d}{8 \cos\alpha} \right)$$

or
$$d = \frac{64}{g} \sin 2\alpha \qquad (1)$$

(*a*) When $h = 0$, the water can follow any physically possible trajectory. It then follows from Eq. (1) that d is maximum when $2\alpha = 90°$

or
$$\alpha = 45° \blacktriangleleft$$

Then
$$d = \frac{64}{9.81} \sin(2 \times 45°)$$

or
$$d_{\max} = 6.52 \text{ m} \blacktriangleleft$$

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PROBLEM 11.116 (Continued)

(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (11.5 \cos \alpha) t$$

At Point B: $x = d_{\max}$ and $t = t_B$

where $$t_B = \frac{11.5}{9.81} \sin 23.8265^\circ = 0.47356 \text{ s}$$

Then $$d_{\max} = (11.5)(\cos 23.8265^\circ)(0.47356)$$

or $$d_{\max} = 4.98 \text{ m} \blacktriangleleft$$

(*b*) From above $$\alpha = 23.8^\circ \blacktriangleleft$$

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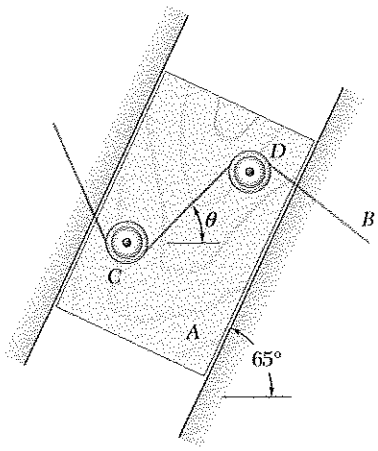
PROBLEM 11.117



As slider block A moves downward at a speed of 0.5 m/s, the velocity with respect to A of the portion of belt B between idler pulleys C and D is $\mathbf{v}_{CD/A} = 2$ m/s $\angle\theta$. Determine the velocity of portion CD of the belt when (*a*) $\theta = 45°$, (*b*) $\theta = 60°$.

SOLUTION

We have $\mathbf{v}_{CD} = \mathbf{v}_A + \mathbf{v}_{CD/A}$

where $\mathbf{v}_A = (0.5 \text{ m/s})(-\cos 65°\mathbf{i} - \sin 65°\mathbf{j})$

$= (0.21131 \text{ m/s})\mathbf{i} - (0.45315 \text{ m/s})\mathbf{j}$

and $\mathbf{v}_{CD/A} = (2 \text{ m/s})(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$

Then $\mathbf{v}_{CD} = [(-0.21131 + 2\cos\theta) \text{ m/s}]\mathbf{i} + [(-0.45315 + 2\sin\theta) \text{ m/s}]\mathbf{j}$

(*a*) We have $\mathbf{v}_{CD} = (-0.21131 + 2\cos 45°)\mathbf{i} + (-0.45315 + 2\sin 45°)\mathbf{j}$

$= (1.20290 \text{ m/s})\mathbf{i} + (0.96106 \text{ m/s})\mathbf{j}$

or $\mathbf{v}_{CD} = 1.540 \text{ m/s} \angle 38.6°$ ◀

(*b*) We have $\mathbf{v}_{CD} = (-0.21131 + 2\cos 60°)\mathbf{i} + (-0.45315 + 2\sin 60°)\mathbf{j}$

$= (0.78869 \text{ m/s})\mathbf{i} + (1.27890 \text{ m/s})\mathbf{j}$

or $\mathbf{v}_{CD} = 1.503 \text{ m/s} \angle 38.3°$ ◀

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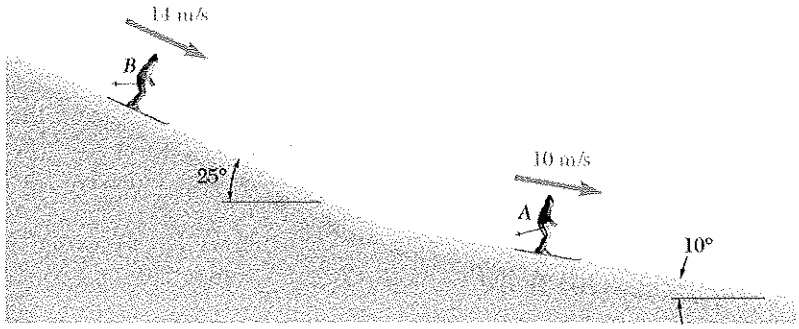
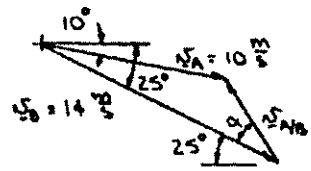


PROBLEM 11.118

The velocities of skiers A and B are as shown. Determine the velocity of A with respect to B.

SOLUTION

We have $$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$



The graphical representation of this equation is then as shown.

Then $$v_{A/B}^2 = 10^2 + 14^2 - 2(10)(14)\cos 15°$$

or $$v_{A/B} = 5.05379 \text{ m/s}$$

and $$\frac{10}{\sin\alpha} = \frac{5.05379}{\sin 15°}$$

or $$\alpha = 30.8°$$

$$\mathbf{v}_{A/B} = 5.05 \text{ m/s} \searrow 55.8° \blacktriangleleft$$

Alternative solution.

$$\begin{aligned}\mathbf{v}_{A/B} &= \mathbf{v}_A - \mathbf{v}_B \\ &= 10\cos 10\mathbf{i} - 10\sin 10\mathbf{j} - (14\cos 25\mathbf{i} - 14\sin 25\mathbf{j}) \\ &= -2.84\mathbf{i} + 4.14\mathbf{j} \\ &= 5.05 \text{ m/s} \searrow 55.8°\end{aligned}$$

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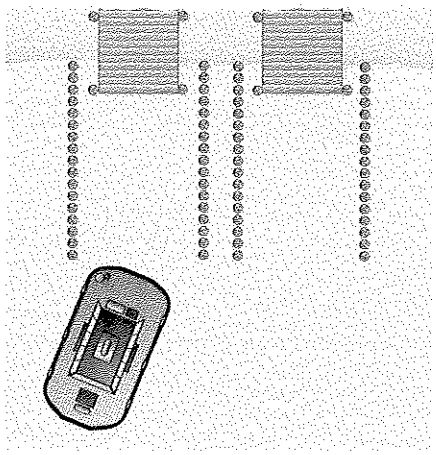


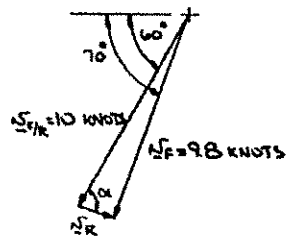
PROBLEM 11.119

Shore-based radar indicates that a ferry leaves its slip with a velocity $\mathbf{v} = 9.8$ knots $\measuredangle 70°$, while instruments aboard the ferry indicate a speed of 10 knots and a heading of 30° west of south relative to the river. Determine the velocity of the river.

SOLUTION

We have $\mathbf{v}_F = \mathbf{v}_R + \mathbf{v}_{F/R}$ or $\mathbf{v}_F = \mathbf{v}_{F/R} + \mathbf{v}_R$

The graphical representation of the second equation is then as shown.



We have $v_R^2 = 9.8^2 + 10^2 - 2(9.8)(10)\cos 10°$

or $v_R = 1.737147$ knots

and $\dfrac{9.8}{\sin\alpha} = \dfrac{1.737\,147}{\sin 10°}$

or $\alpha = 78.41°$

Noting that

$\mathbf{v}_R = 1.737$ knots $\measuredangle 18.41°$ ◀

Alternatively one could use vector algebra.

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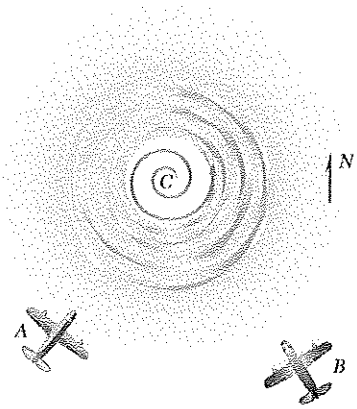
PROBLEM 11.120



Airplanes A and B are flying at the same altitude and are tracking the eye of hurricane C. The relative velocity of C with respect to A is $\mathbf{v}_{C/A} = 235$ mi/h $\measuredangle 75°$, and the relative velocity of C with respect to B is $\mathbf{v}_{C/B} = 260$ mi/h $\measuredangle 40°$. Determine (a) the relative velocity of B with respect to A, (b) the velocity of A if ground-based radar indicates that the hurricane is moving at a speed of 24 mi/h due north, (c) the change in position of C with respect to B during a 15-min interval.

SOLUTION

(a) We have $\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$

and $\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

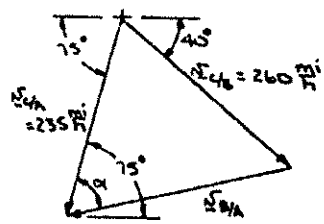
Then $\mathbf{v}_A + \mathbf{v}_{C/A} = \mathbf{v}_B + \mathbf{v}_{C/B}$

or $\mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{C/A} - \mathbf{v}_{C/B}$

Now $\mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{B/A}$

so that $\mathbf{v}_{B/A} = \mathbf{v}_{C/A} - \mathbf{v}_{C/B}$

or $\mathbf{v}_{C/A} = \mathbf{v}_{C/B} + \mathbf{v}_{B/A}$



The graphical representation of the last equation is then as shown.

We have $v_{B/A}^2 = 235^2 + 260^2 - 2(235)(260)\cos 65°$

or $v_{B/A} = 266.798$ mi/h

and $\dfrac{260}{\sin\alpha} = \dfrac{266.798}{\sin 65°}$

or $\alpha = 62.03°$

$\mathbf{v}_{B/A} = 267$ mi/h $\measuredangle 12.97°$ ◀

(b) We have $\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$

or $\mathbf{v}_A = (24 \text{ mi/h})\mathbf{j} - (235 \text{ mi/h})(-\cos 75°\mathbf{i} - \sin 75°\mathbf{j})$

$\mathbf{v}_A = (60.822 \text{ mi/h})\mathbf{i} + (250.99 \text{ mi/h})\mathbf{j}$

or $\mathbf{v}_A = 258$ mi/h $\measuredangle 76.4°$ ◀

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PROBLEM 11.120 (Continued)

(*c*) Noting that the velocities of B and C are constant. We have

$$\mathbf{r}_B = (\mathbf{r}_B)_0 + \mathbf{v}_B t \qquad \mathbf{r}_C = (\mathbf{r}_C)_0 + \mathbf{v}_C t$$

Now

$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = [(\mathbf{r}_C)_0 - (\mathbf{r}_B)_0] + (\mathbf{v}_C - \mathbf{v}_B)t$$

$$= [(\mathbf{r}_C)_0 - (\mathbf{r}_B)_0] + \mathbf{v}_{C/B} t$$

Then

$$\Delta\mathbf{r}_{C/B} = (\mathbf{r}_{C/B})_{t_2} - (\mathbf{r}_{C/B})_{t_1} = \mathbf{v}_{C/B}(t_2 - t_1) = \mathbf{v}_{C/B}\Delta t$$

For $\Delta t = 15$ min:

$$\Delta r_{C/B} = (260 \text{ mi/h})\left(\frac{1}{4}\text{h}\right) = 65 \text{ mi}$$

$\Delta\mathbf{r}_{C/B} = 65$ mi $\searrow$ 40° ◀

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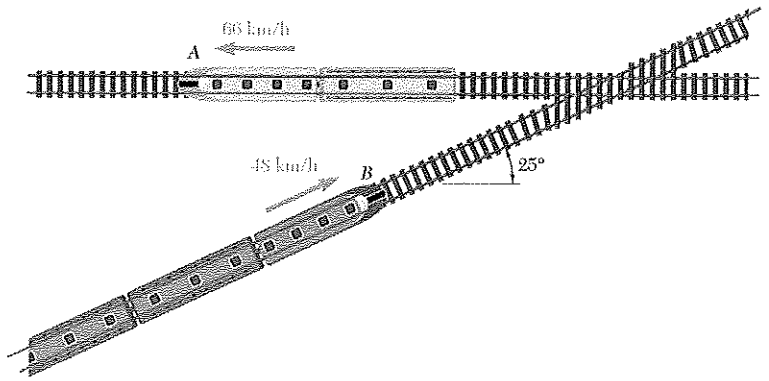


PROBLEM 11.121

The velocities of commuter trains A and B are as shown. Knowing that the speed of each train is constant and that B reaches the crossing 10 min after A passed through the same crossing, determine (*a*) the relative velocity of B with respect to A, (*b*) the distance between the fronts of the engines 3 min after A passed through the crossing.

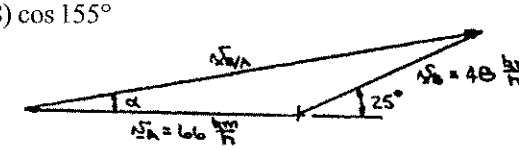
SOLUTION

(*a*) We have $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown.

Then $v_{B/A}^2 = 66^2 + 48^2 - 2(66)(48)\cos 155°$

or $v_{B/A} = 111.366$ km/h



and $\dfrac{48}{\sin\alpha} = \dfrac{111.366}{\sin 155°}$

or $\alpha = 10.50°$

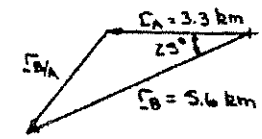
$\mathbf{v}_{B/A} = 111.4$ km/h $\measuredangle$ 10.50° ◀

(*b*) First note that

at $t = 3$ min, A is $(66 \text{ km/h})\left(\frac{3}{60}\right) = 3.3$ km west of the crossing.

at $t = 3$ min, B is $(48 \text{ km/h})\left(\frac{7}{60}\right) = 5.6$ km southwest of the crossing.

Now $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$



Then at $t = 3$ min, we have

$$r_{B/A}^2 = 3.3^2 + 5.6^2 - 2(3.3)(5.6)\cos 25°$$

or $r_{B/A} = 2.96$ km ◀

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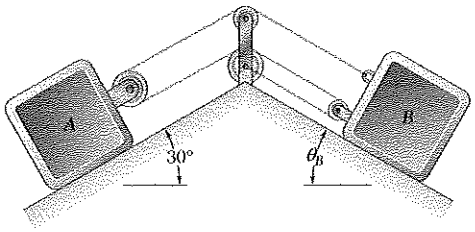


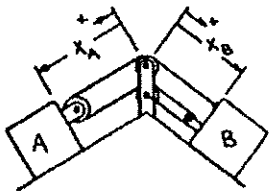
PROBLEM 11.122

Knowing that the velocity of block B with respect to block A is $\mathbf{v}_{B/A} = 5.6$ m/s $\measuredangle$ 70°, determine the velocities of A and B.

SOLUTION

From the diagram

$$2x_A + 3x_B = \text{constant}$$

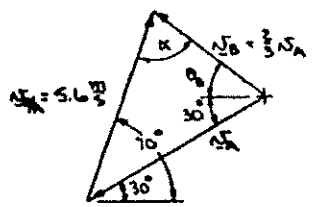


Then
$$2v_A + 3v_B = 0$$

or
$$|v_B| = \frac{2}{3}v_A$$

Now
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

and noting that $\mathbf{v}_A$ and $\mathbf{v}_B$ must be parallel to surfaces A and B, respectively, the graphical representation of this equation is then as shown. *Note:* Assuming that $\mathbf{v}_A$ is directed up the incline leads to a velocity diagram that does not "close."



First note
$$\alpha = 180° - (40° + 30° + \theta_B)$$
$$= 110° - \theta_B$$

Then
$$\frac{v_A}{\sin(110° - \theta_B)} = \frac{\frac{2}{3}v_A}{\sin 40°} = \frac{5.6}{\sin(30° + \theta_B)}$$

or
$$v_A \sin 40° = \frac{2}{3}v_A \sin(110° - \theta_B)$$

or
$$\sin(110° - \theta_B) = 0.96418$$

or
$$\theta_B = 35.3817°$$

and
$$\theta_B = 4.6183°$$

For $\theta_B = 35.3817°$:
$$v_B = \frac{2}{3}v_A = \frac{5.6 \sin 40°}{\sin(30° + 35.3817°)}$$

or
$$v_A = 5.94 \text{ m/s} \quad v_B = 3.96 \text{ m/s}$$

$\mathbf{v}_A = 5.94$ m/s $\measuredangle$ 30° ◀

$\mathbf{v}_B = 3.96$ m/s $\measuredangle$ 35.4° ◀

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PROBLEM 11.122 (Continued)

For $\theta_B = 4.6183°$:

$$v_B = \frac{2}{3} v_A = \frac{5.6 \sin 40°}{\sin (30° + 4.6183°)}$$

or

$$v_A = 9.50 \text{ m/s}$$

$$v_B = 6.34 \text{ m/s}$$

$\mathbf{v}_A = 9.50$ m/s $\nearrow 30°$ ◀

$\mathbf{v}_B = 6.34$ m/s $\searrow 4.62°$ ◀

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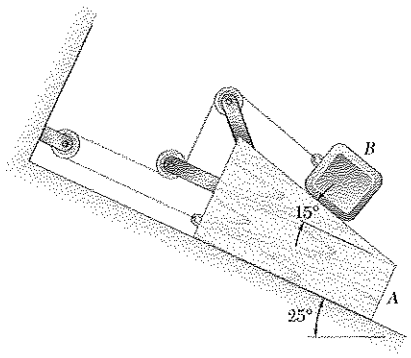


PROBLEM 11.123

Knowing that at the instant shown block A has a velocity of 8 in./s and an acceleration of 6 in./s^2 both directed down the incline, determine (a) the velocity of block B, (b) the acceleration of block B.

SOLUTION

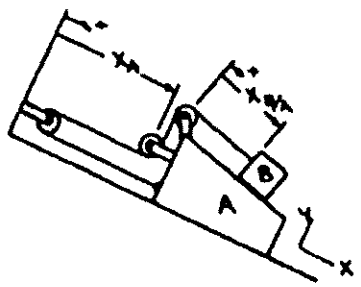
From the diagram $\quad 2x_A + x_{B/A} = \text{constant}$

Then $\quad 2v_A + v_{B/A} = 0$

or $\quad |v_{B/A}| = 16 \text{ in./s}$

and $\quad 2a_A + a_{B/A} = 0$

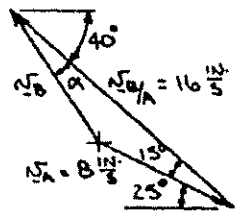
or $\quad |a_{B/A}| = 12 \text{ in./s}^2$



Note that $\mathbf{v}_{B/A}$ and $\mathbf{a}_{B/A}$ must be parallel to the top surface of block A.

(a) We have $\quad \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown. Note that because A is moving downward, B must be moving upward relative to A.



We have $\quad v_B^2 = 8^2 + 16^2 - 2(8)(16)\cos 15°$

or $\quad v_B = 8.5278 \text{ in./s}$

and $\quad \dfrac{8}{\sin\alpha} = \dfrac{8.5278}{\sin 15°}$

or $\quad \alpha = 14.05°$

$\mathbf{v}_B = 8.53 \text{ in./s} \measuredangle 54.1°$ ◀

(b) The same technique that was used to determine $\mathbf{v}_B$ can be used to determine $\mathbf{a}_B$. An alternative method is as follows.

We have $\quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$$= (6\mathbf{i}) + 12(-\cos 15°\mathbf{i} + \sin 15°\mathbf{j})^*$$

$$= -(5.5911 \text{ in./s}^2)\mathbf{i} + (3.1058 \text{ in./s}^2)\mathbf{j}$$

or $\quad \mathbf{a}_B = 6.40 \text{ in./s}^2 \measuredangle 54.1°$ ◀

* Note the orientation of the coordinate axes on the sketch of the system.

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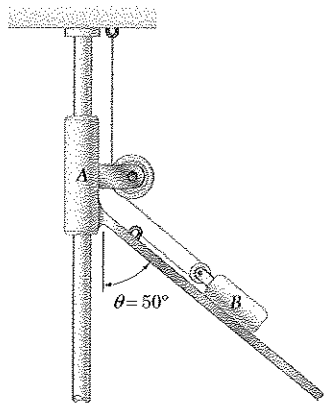
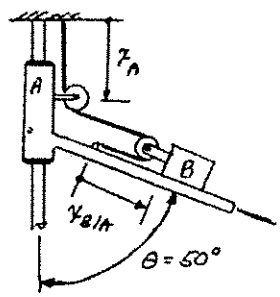


PROBLEM 11.124

Knowing that at the instant shown assembly A has a velocity of 9 in./s and an acceleration of 15 in./s^2 both directed downward, determine (a) the velocity of block B, (b) the acceleration of block B.

SOLUTION



Length of cable = constant

$$L = x_A + 2x_{B/A} = \text{constant}$$

$$v_A + 2v_{B/A} = 0 \tag{1}$$

$$a_A + 2a_{B/A} = 0 \tag{2}$$

Data:

$$\mathbf{a}_A = 15 \text{ in./s}^2 \downarrow$$

$$\mathbf{v}_A = 9 \text{ in./s} \downarrow$$

Eqs. (1) and (2)

$$a_A = -2a_{B/A} \qquad v_A = -2v_{B/A}$$

$$15 = -2a_{B/A} \qquad 9 = -2v_{B/A}$$

$$a_{B/A} = -7.5 \text{ in./s}^2 \qquad v_{B/A} = -4.5 \text{ in./s}$$

$$\mathbf{a}_{B/A} = 7.5 \text{ in./s}^2 \measuredangle 40° \qquad \mathbf{v}_{B/A} = 4.5 \text{ in./s} \measuredangle 40°$$

(a) Velocity of B.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

Law of cosines:

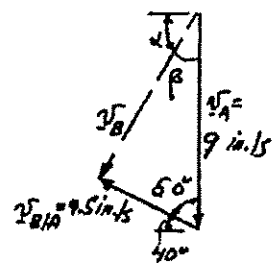
$$v_B^2 = (9)^2 + (4.5)^2 - 2(9)(4.5)\cos 50°$$

$$v_B = 7.013 \text{ in./s}$$

Law of sines:

$$\frac{\sin\beta}{4.5} = \frac{\sin 50°}{7.013} \qquad \beta = 29.44°$$

$$\alpha = 90° - \beta = 90° - 29.44° = 60.56°$$



$$\mathbf{v}_B = 7.01 \text{ in./s} \measuredangle 60.6° \blacktriangleleft$$

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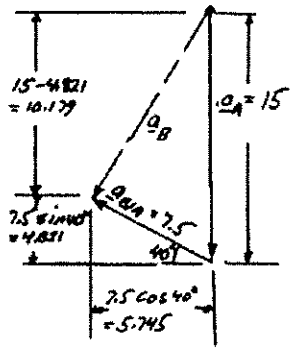
PROBLEM 11.124 (Continued)

(*b*) Acceleration of *B*. $\mathbf{a}_B$ may be found by using analysis similar to that used above for v_B. An alternate method is



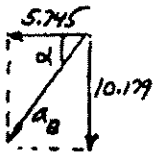
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\mathbf{a}_B = 15 \text{ in./s}^2 \downarrow + 7.5 \text{ in./s}^2 \measuredangle 40°$$

$$= -15\mathbf{j} - (7.5 \cos 40°)\mathbf{i} + (7.5 \sin 40°)\mathbf{j}$$

$$= -15\mathbf{j} - 5.745\mathbf{i} + 4.821\mathbf{j}$$

$$\mathbf{a}_B = -5.745\mathbf{i} - 10.179\mathbf{j}$$



$$\mathbf{a}_B = 11.69 \text{ in./s}^2 \measuredangle 60.6° \blacktriangleleft$$

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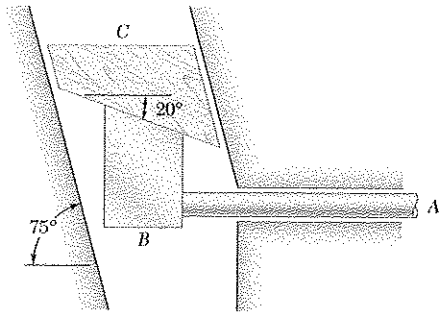


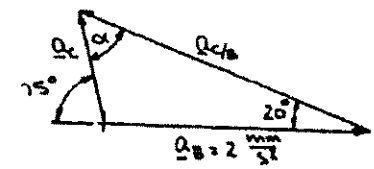
PROBLEM 11.125

The assembly of rod A and wedge B starts from rest and moves to the right with a constant acceleration of 2 mm/s^2. Determine (a) the acceleration of wedge C, (b) the velocity of wedge C when $t = 10$ s.

SOLUTION

(a) We have $$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$$

The graphical representation of this equation is then as shown.



First note $$\alpha = 180° - (20° + 105°)$$
$$= 55°$$

Then $$\frac{a_C}{\sin 20°} = \frac{2}{\sin 55°}$$
$$a_C = 0.83506 \text{ mm/s}^2$$

$$\mathbf{a}_C = 0.835 \text{ mm/s}^2 \measuredangle 75° \blacktriangleleft$$

(b) For uniformly accelerated motion

$$v_C = 0 + a_C t$$

At $t = 10$ s: $$v_C = (0.83506 \text{ mm/s}^2)(10 \text{ s})$$
$$= 8.3506 \text{ mm/s}$$

or $$\mathbf{v}_C = 8.35 \text{ mm/s} \measuredangle 75° \blacktriangleleft$$

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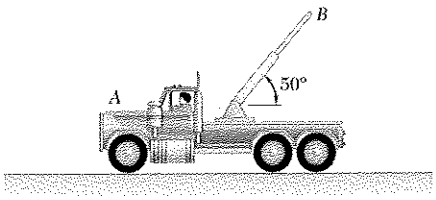


PROBLEM 11.126

As the truck shown begins to back up with a constant acceleration of 1.2 m/s^2, the outer section B of its boom starts to retract with a constant acceleration of 0.5 m/s^2 relative to the truck. Determine (a) the acceleration of section B, (b) the velocity of section B when $t = 2$ s.

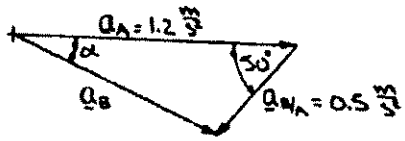
SOLUTION

(a) We have $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

The graphical representation of this equation is then as shown.

We have $a_B^2 = 1.2^2 + 0.5^2 - 2(1.2)(0.5)\cos 50°$

or $a_B = 0.95846$ m/s^2



and $\dfrac{0.5}{\sin\alpha} = \dfrac{0.95846}{\sin 50°}$

or $\alpha = 23.6°$

$\mathbf{a}_B = 0.958$ m/s^2 ∠ 23.6° ◀

(b) For uniformly accelerated motion

$v_B = 0 + a_B t$

At $t = 2$ s: $v_B = (0.95846 \text{ m/s}^2)(2 \text{ s})$

$= 1.91692$ m/s

or $\mathbf{v}_B = 1.917$ m/s ∠ 23.6° ◀

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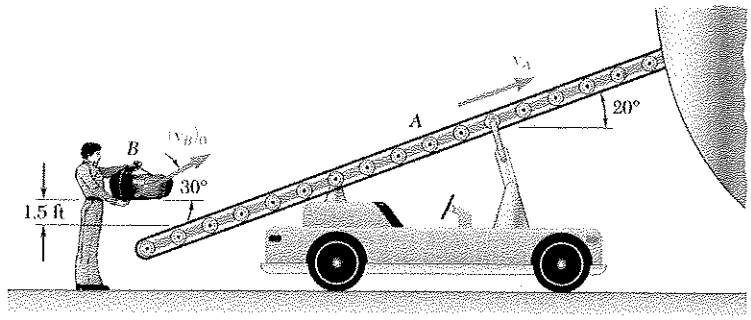
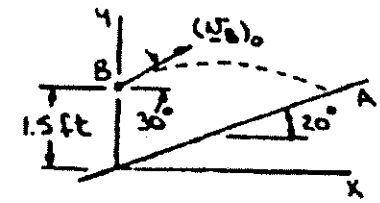


PROBLEM 11.127

Conveyor belt A, which forms a 20° angle with the horizontal, moves at a constant speed of 4 ft/s and is used to load an airplane. Knowing that a worker tosses duffel bag B with an initial velocity of 2.5 ft/s at an angle of 30° with the horizontal, determine the velocity of the bag relative to the belt as it lands on the belt.

SOLUTION

First determine the velocity of the bag as it lands on the belt. Now



$$[(v_B)_x]_0 = (v_B)_0 \cos 30°$$
$$= (2.5 \text{ ft/s}) \cos 30°$$
$$[(v_B)_y]_0 = (v_B)_0 \sin 30°$$
$$= (2.5 \text{ ft/s}) \sin 30°$$

Horizontal motion. (Uniform)

$$x = 0 + [(v_B)_x]_0 t \qquad (v_B)_x = [(v_B)_x]_0$$
$$= (2.5 \cos 30°)t \qquad = 2.5 \cos 30°$$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + [(v_B)_y]_0 t - \frac{1}{2} g t^2 \qquad (v_B)_y = [(v_B)_y]_0 - gt$$
$$= 1.5 + (2.5 \sin 30°)t - \frac{1}{2} g t^2 \qquad = 2.5 \sin 30° - gt$$

The equation of the line collinear with the top surface of the belt is

$$y = x \tan 20°$$

Thus, when the bag reaches the belt

$$1.5 + (2.5 \sin 30°)t - \frac{1}{2} g t^2 = [(2.5 \cos 30°)t] \tan 20°$$

or
$$\frac{1}{2}(32.2)t^2 + 2.5(\cos 30° \tan 20° - \sin 30°)t - 1.5 = 0$$

or
$$16.1t^2 - 0.46198t - 1.5 = 0$$

Solving $t = 0.31992$ s and $t = -0.29122$ s (Reject)

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PROBLEM 11.127 (Continued)

The velocity $\mathbf{v}_B$ of the bag as it lands on the belt is then

$$\mathbf{v}_B = (2.5 \cos 30°)\mathbf{i} + [2.5 \sin 30° - 32.2(0.319\,92)]\mathbf{j}$$
$$= (2.1651 \text{ ft/s})\mathbf{i} - (9.0514 \text{ ft/s})\mathbf{j}$$

Finally $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

or $\mathbf{v}_{B/A} = (2.1651\mathbf{i} - 9.0514\mathbf{j}) - 4(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$
$= -(1.59367 \text{ ft/s})\mathbf{i} - (10.4195 \text{ ft/s})\mathbf{j}$

or $\mathbf{v}_{B/A} = 10.54 \text{ ft/s} \measuredangle 81.3°$ ◀

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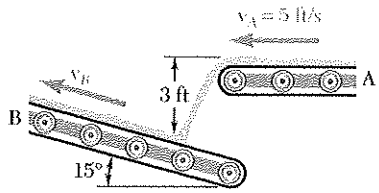

PROBLEM 11.128

Determine the required velocity of the belt B if the relative velocity with which the sand hits belt B is to be (a) vertical, (b) as small as possible.

SOLUTION

A grain of sand will undergo projectile motion.

$$v_{S_x} = v_{S_{x_0}} = \text{constant} = -5 \text{ ft/s}$$

y-direction. $$v_{S_y} = \sqrt{2gh} = \sqrt{(2)(32.2 \text{ ft/s}^2)(3 \text{ ft})} = 13.90 \text{ ft/s} \downarrow$$

Relative velocity. $$\mathbf{v}_{S/B} = \mathbf{v}_S - \mathbf{v}_B \qquad (1)$$

(a) If $v_{S/B}$ is vertical,

$$-v_{S/B}\mathbf{j} = -5\mathbf{i} - 13.9\mathbf{j} - (-v_B \cos 15\mathbf{i} + v_B \sin 15\mathbf{j})$$
$$= -5\mathbf{i} - 13.9\mathbf{j} + v_B \cos 15\mathbf{i} - v_B \sin 15\mathbf{j}$$

Equate components. $$\mathbf{i}: \quad 0 = -5 + v_B \cos 15^\circ \qquad v_B = \frac{5}{\cos 15^\circ} = 5.176 \text{ ft/s}$$

$$\mathbf{v}_B = 5.18 \text{ ft/s} \measuredangle 15^\circ \blacktriangleleft$$

(b) $v_{S/C}$ is as small as possible, so make $v_{S/B} \perp$ to v_B into (1).

$$-v_{S/B} \sin 15^\circ \mathbf{i} - v_{S/B} \cos 15^\circ \mathbf{j} = -5\mathbf{i} - 13.9\mathbf{j} + v_B \cos 15^\circ \mathbf{i} - v_B \sin 15^\circ \mathbf{j}$$

Equate components and transpose terms.

$$(\sin 15^\circ)\, v_{S/B} + (\cos 15^\circ)\, v_B = 5$$
$$(\cos 15^\circ)\, v_{S/B} - (\sin 15^\circ)\, v_B = 13.90$$

Solving, $$v_{S/B} = 14.72 \text{ ft/s} \qquad v_B = 1.232 \text{ ft/s}$$

$$\mathbf{v}_B = 1.232 \text{ ft/s} \measuredangle 15^\circ \blacktriangleleft$$

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PROBLEM 11.129

As observed from a ship moving due east at 9 km/h, the wind appears to blow from the south. After the ship has changed course and speed, and as it is moving due north at 6 km/h, the wind appears to blow from the southwest. Assuming that the wind velocity is constant during the period of observation, determine the magnitude and direction of the true wind velocity.

SOLUTION

$$\mathbf{v}_{\text{wind}} = \mathbf{v}_{\text{ship}} + \mathbf{v}_{\text{wind/ship}}$$

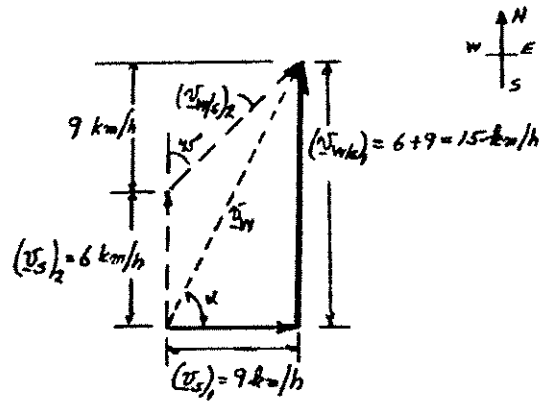
$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$

Case ①

$$\mathbf{v}_s = 9 \text{ km/h} \rightarrow; \quad \mathbf{v}_{w/s} \uparrow$$

Case ②

$$\mathbf{v}_s = 6 \text{ km/h} \uparrow; \quad \mathbf{v}_{w/s} \nearrow$$



$$\tan\alpha = \frac{15}{9} = 1.6667$$

$$\alpha = 59.0°$$

$$v_w = \sqrt{9^2 + 15^2} = 17.49 \text{ km/h}$$

$$\mathbf{v}_w = 17.49 \text{ km/h} \measuredangle 59.0° \blacktriangleleft$$

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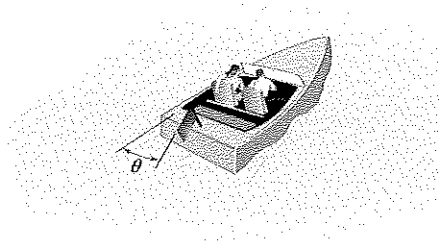
PROBLEM 11.130



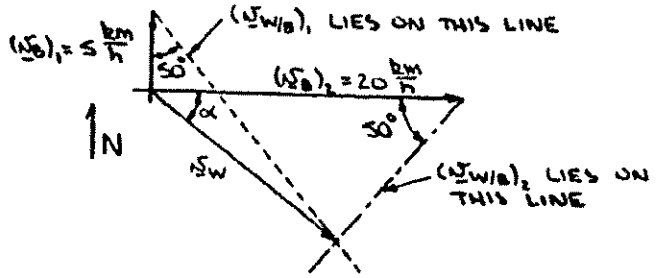
When a small boat travels north at 5 km/h, a flag mounted on its stern forms an angle $\theta = 50°$ with the centerline of the boat as shown. A short time later, when the boat travels east at 20 km/h, angle θ is again 50°. Determine the speed and the direction of the wind.

SOLUTION

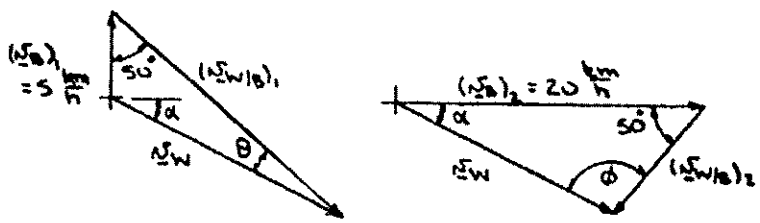
We have

$$\mathbf{v}_W = \mathbf{v}_B + \mathbf{v}_{W/B}$$

Using this equation, the two cases are then graphically represented as shown.



With $\mathbf{v}_W$ now defined, the above diagram is redrawn for the two cases for clarity.



Noting that

$$\theta = 180° - (50° + 90° + \alpha) \qquad \phi = 180° - (50° + \alpha)$$
$$= 40° - \alpha \qquad = 130° - \alpha$$

We have

$$\frac{v_W}{\sin 50°} = \frac{5}{\sin (40° - \alpha)} \qquad \frac{v_W}{\sin 50°} = \frac{20}{\sin (130° - \alpha)}$$

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PROBLEM 11.130 (Continued)

Therefore $$\frac{5}{\sin(40^\circ-\alpha)}=\frac{20}{\sin(130^\circ-\alpha)}$$

or $$\sin 130^\circ\cos\alpha-\cos 130^\circ\sin\alpha=4(\sin 40^\circ\cos\alpha-\cos 40^\circ\sin\alpha)$$

or $$\tan\alpha=\frac{\sin 130^\circ-4\sin 40^\circ}{\cos 130^\circ-4\cos 40^\circ}$$

or $$\alpha=25.964^\circ$$

Then $$v_W=\frac{5\sin 50^\circ}{\sin(40^\circ-25.964^\circ)}=15.79 \text{ km/h}$$

$\mathbf{v}_W = 15.79$ km/h $\measuredangle 26.0^\circ$ ◀

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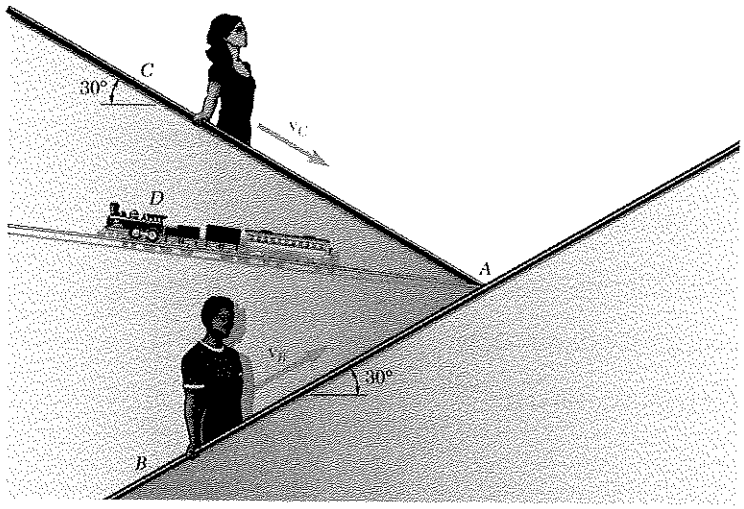


PROBLEM 11.131

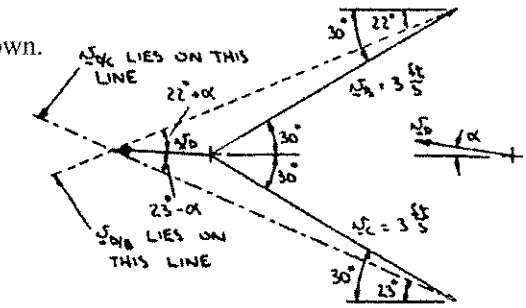
As part of a department store display, a model train D runs on a slight incline between the store's up and down escalators. When the train and shoppers pass Point A, the train appears to a shopper on the up escalator B to move downward at an angle of 22° with the horizontal, and to a shopper on the down escalator C to move upward at an angle of 23° with the horizontal and to travel to the left. Knowing that the speed of the escalators is 3 ft/s, determine the speed and the direction of the train.

SOLUTION

We have

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$
$$\mathbf{v}_D = \mathbf{v}_C + \mathbf{v}_{D/C}$$

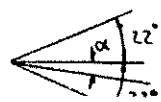
The graphical representations of these equations are then as shown.



Then $\dfrac{v_D}{\sin 8°} = \dfrac{3}{\sin(22° + \alpha)}$ $\dfrac{v_D}{\sin 7°} = \dfrac{3}{\sin(23° - \alpha)}$

Equating the expressions for $\dfrac{v_D}{3}$

$$\frac{\sin 8°}{\sin(22° + \alpha)} = \frac{\sin 7°}{\sin(23° - \alpha)}$$



or $\sin 8°(\sin 23° \cos\alpha - \cos 23° \sin\alpha) = \sin 7°(\sin 22° \cos\alpha + \cos 22° \sin\alpha)$

or $$\tan\alpha = \frac{\sin 8° \sin 23° - \sin 7° \sin 22°}{\sin 8° \cos 23° + \sin 7° \cos 22°}$$

or $$\alpha = 2.0728°$$

Then $$v_D = \frac{3 \sin 8°}{\sin(22° + 2.0728°)} = 1.024 \text{ ft/s}$$

$$\mathbf{v}_D = 1.024 \text{ ft/s} \searrow 2.07° \blacktriangleleft$$

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PROBLEM 11.131 (Continued)

Alternate solution using components.

$$\mathbf{v}_B = (3 \text{ ft/s}) \angle 30° = (2.5981 \text{ ft/s})\mathbf{i} + (1.5 \text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_C = (3 \text{ ft/s}) \angle 30° = (2.5981 \text{ ft/s})\mathbf{i} - (1.5 \text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_{D/B} = u_1 \angle 22° = -(u_1 \cos 22°)\mathbf{i} - (u_1 \sin 22°)\mathbf{j}$$

$$\mathbf{v}_{D/C} = u_2 \angle 23° = -(u_2 \cos 23°)\mathbf{i} + (u_2 \sin 23°)\mathbf{j}$$

$$\mathbf{v}_D = v_D \angle \alpha = -(v_D \cos \alpha)\mathbf{i} + (v_D \sin \alpha)\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = \mathbf{v}_C + \mathbf{v}_{D/C}$$

$$2.5981\mathbf{i} + 1.5\mathbf{j} - (u_1 \cos 22°)\mathbf{i} - (u_1 \sin 22°)\mathbf{j} = 2.5981\mathbf{i} - 1.5\mathbf{j} - (u_2 \cos 23°)\mathbf{i} + (u_2 \sin 23°)\mathbf{j}$$

Separate into components, and transpose and change

$$u_1 \cos 22° - u_2 \cos 23° = 0$$

$$u_1 \sin 22° + u_1 \sin 23° = 3$$

Solving for u_1 and u_2, $\quad u_1 = 3.9054$ ft/s $\qquad u_2 = 3.9337$ ft/s

$$\mathbf{v}_D = 2.5981\mathbf{i} + 1.5\mathbf{j} - (3.9054 \cos 22°)\mathbf{i} - (3.9054 \sin 22°)\mathbf{j}$$
$$= -(1.0229 \text{ ft/s})\mathbf{i} + (0.0370 \text{ ft/s})\mathbf{j}$$

or

$$\mathbf{v}_D = 2.5981\mathbf{i} - 1.5\mathbf{j} - (3.9337 \cos 23°)\mathbf{i} + (3.9337 \sin 23°)\mathbf{j}$$
$$= -(1.0229 \text{ ft/s})\mathbf{i} + (0.0370 \text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_D = 1.024 \text{ ft/s} \angle 2.07° \blacktriangleleft$$

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PROBLEM 11.132

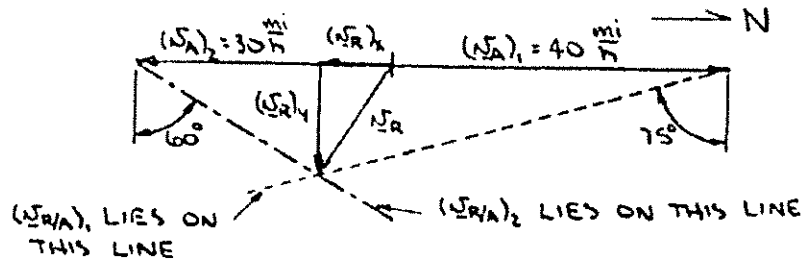
The paths of raindrops during a storm appear to form an angle of 75° with the vertical and to be directed to the left when observed through a left-side window of an automobile traveling north at a speed of 40 mi/h. When observed through a right-side window of an automobile traveling south at a speed of 30 mi/h, the raindrops appear to form an angle of 60° with the vertical. If the driver of the automobile traveling north were to stop, at what angle and with what speed would she observe the drops to fall?

SOLUTION

We have

$$\mathbf{v}_R = (\mathbf{v}_A)_1 + (\mathbf{v}_{R/A})_1$$
$$\mathbf{v}_R = (\mathbf{v}_A)_2 + (\mathbf{v}_{R/A})_2$$

The graphical representations of these equations are then as shown. Note that the line of action of $(\mathbf{v}_{R/A})_2$ must be directed as shown so that the second velocity diagram "closes."



From the diagram

$$(v_R)_y = [40 + (v_R)_x]\tan 15°$$

and

$$(v_R)_y = [30 - (v_R)_x]\tan 30°$$

Equating the expressions for $(v_R)_y$

$$[40 + (v_R)_x]\tan 15° = [30 - (v_R)_x]\tan 30°$$

or

$$(v_R)_x = 7.8109 \text{ mi/h}$$

Then

$$(v_R)_y = (40 + 7.8109)\tan 15° = 12.8109 \text{ mi/h}$$

$$\mathbf{v}_R = 15.00 \text{ mi/h} \measuredangle 58.6° \blacktriangleleft$$

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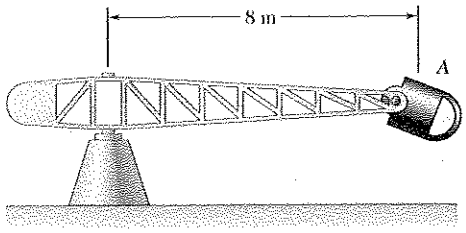

PROBLEM 11.133

Determine the peripheral speed of the centrifuge test cab A for which the normal component of the acceleration is $10g$.

SOLUTION

$$a_n = 10g = 10(9.81 \text{ m/s}^2) = 98.1 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} \qquad v^2 = \rho a_n = (8 \text{ m})(98.1 \text{ m/s}^2) = 784.8 \text{ m}^2/\text{s}^2$$

$$v = 28.0 \text{ m/s} \blacktriangleleft$$

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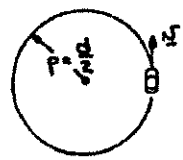
PROBLEM 11.134

To test its performance, an automobile is driven around a circular test track of diameter d. Determine (a) the value of d if when the speed of the automobile is 72 km/h, the normal component of the acceleration is 3.2 m/s^2, (b) the speed of the automobile if $d = 180$ m and the normal component of the acceleration is measured to be 0.6 g.

SOLUTION

(a) First note $v = 72 \text{ km/h} = 20 \text{ m/s}$



Now $a_n = \dfrac{v^2}{\rho}$

or $\dfrac{d}{2} = \dfrac{(20 \text{ m/s})^2}{3.2 \text{ m/s}^2}$

or $d = 250 \text{ m}$ ◀

(b) We have $a_n = \dfrac{v^2}{\rho}$

Then $v^2 = (0.6 \times 9.81 \text{ m/s}^2)\left(\dfrac{1}{2} \times 180 \text{ m}\right)$

or $v = 23.016 \text{ m/s}$

or $v = 82.9 \text{ km/h}$ ◀

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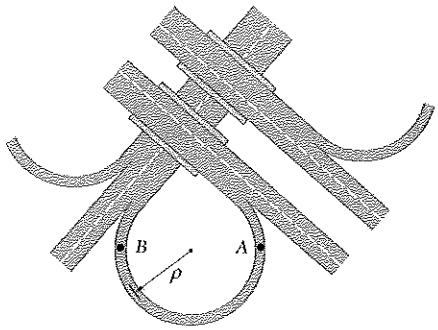

PROBLEM 11.135

Determine the smallest radius that should be used for a highway if the normal component of the acceleration of a car traveling at 45 mi/h is not to exceed 2.4 ft/s^2.

SOLUTION

$$a_n = \frac{v^2}{\rho} \qquad a_n = 2.4 \text{ ft/s}^2$$

$$v = 45 \text{ mi/h} = 66 \text{ ft/s}$$

$$2.4 \text{ ft/s}^2 = \frac{(66 \text{ ft/s})^2}{\rho} \qquad \rho = 1815 \text{ ft} \blacktriangleleft$$

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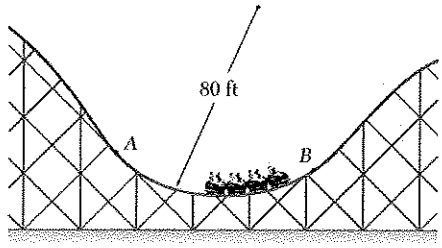

PROBLEM 11.136

Determine the maximum speed that the cars of the roller-coaster can reach along the circular portion AB of the track if the normal component of their acceleration cannot exceed 3 g.

SOLUTION

We have $$a_n = \frac{v^2}{\rho}$$

Then $$(v_{\max})_{AB}^2 = (3 \times 32.2 \text{ ft/s}^2)(80 \text{ ft})$$

or $$(v_{\max})_{AB} = 87.909 \text{ ft/s}$$

or $$(v_{\max})_{AB} = 59.9 \text{ mi/h} \blacktriangleleft$$

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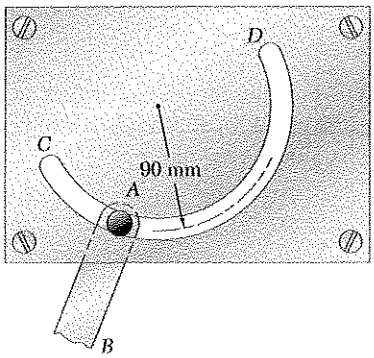

PROBLEM 11.137

Pin A, which is attached to link AB, is constrained to move in the circular slot CD. Knowing that at $t = 0$ the pin starts from rest and moves so that its speed increases at a constant rate of 20 mm/s^2, determine the magnitude of its total acceleration when (a) $t = 0$, (b) $t = 2$ s.

SOLUTION

(a) At $t = 0$, $v_A = 0$, which implies $(a_A)_n = 0$

$$a_A = (a_A)_t$$

or $$a_A = 20 \text{ mm/s}^2 \blacktriangleleft$$

(b) We have uniformly accelerated motion

$$v_A = 0 + (a_A)_t \, t$$

At $t = 2$ s: $$v_A = (20 \text{ mm/s}^2)(2 \text{ s}) = 40 \text{ mm/s}$$

Now $$(a_A)_n = \frac{v_A^2}{\rho_A} = \frac{(40 \text{ mm/s})^2}{90 \text{ mm}} = 17.7778 \text{ mm/s}^2$$

Finally, $$a_A^2 = (a_A)_t^2 + (a_A)_n^2$$
$$= (20)^2 + (17.7778)^2$$

or $$a_A = 26.8 \text{ mm/s}^2 \blacktriangleleft$$

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PROBLEM 11.138

A monorail train starts from rest on a curve of radius 400 m and accelerates at the constant rate a_t. If the maximum total acceleration of the train must not exceed 1.5 m/s^2, determine (*a*) the shortest distance in which the train can reach a speed of 72 km/h, (*b*) the corresponding constant rate of acceleration a_t.

SOLUTION

When $v = 72$ km/h $= 20$ m/s and $\rho = 400$ m,

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{400} = 1.000 \text{ m/s}^2$$

But $$a = \sqrt{a_n^2 + a_t^2}$$

$$a_t = \sqrt{a^2 - a_n^2} = \sqrt{(1.5)^2 - (1.000)^2} = \pm 1.11803 \text{ m/s}^2$$

Since the train is accelerating, reject the negative value.

(*a*) Distance to reach the speed.

$$v_0 = 0$$

Let $$x_0 = 0$$

$$v_1^2 = v_0^2 + 2a_t(x_1 - x_0) = 2a_t x_1$$

$$x_1 = \frac{v_1^2}{2a_t} = \frac{(20)^2}{(2)(1.11803)} \qquad x_1 = 178.9 \text{ m} \blacktriangleleft$$

(*b*) Corresponding tangential acceleration.

$$a_t = 1.118 \text{ m/s}^2 \blacktriangleleft$$

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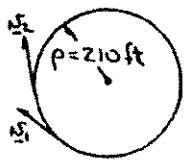
PROBLEM 11.139

An outdoor track is 420 ft in diameter. A runner increases her speed at a constant rate from 14 to 24 ft/s over a distance of 95 ft. Determine the total acceleration of the runner 2 s after she begins to increase her speed.

SOLUTION

We have uniformly accelerated motion



$$v_2^2 = v_1^2 + 2a_t\Delta s_{12}$$

Substituting $(24 \text{ ft/s})^2 = (14 \text{ ft/s})^2 + 2a_t(95 \text{ ft})$

or $a_t = 2 \text{ ft/s}^2$

Also $v = v_1 + a_t t$

At $t = 2$ s: $v = 14 \text{ ft/s} + (2 \text{ ft/s}^2)(2 \text{ s}) = 18 \text{ ft/s}$

Now $a_n = \dfrac{v^2}{\rho}$

At $t = 2$ s: $a_n = \dfrac{(18 \text{ ft/s})^2}{210 \text{ ft}} = 1.54286 \text{ ft/s}^2$

Finally $a^2 = a_t^2 + a_n^2$

At $t = 2$ s: $a^2 = 2^2 + 1.54286^2$

or $a = 2.53 \text{ ft/s}^2$ ◀

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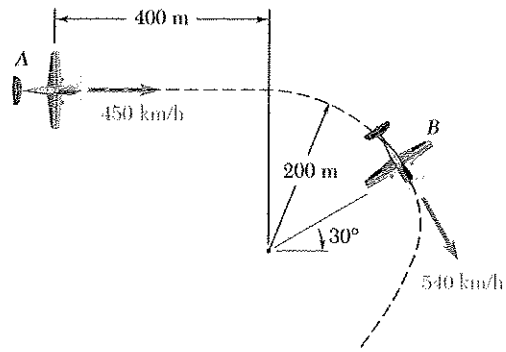


PROBLEM 11.140

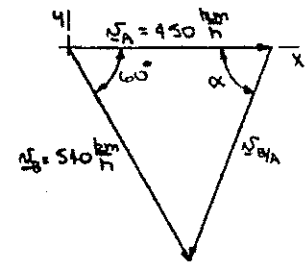
At a given instant in an airplane race, airplane A is flying horizontally in a straight line, and its speed is being increased at the rate of 8 m/s^2. Airplane B is flying at the same altitude as airplane A and, as it rounds a pylon, is following a circular path of 300-m radius. Knowing that at the given instant the speed of B is being decreased at the rate of 3 m/s^2, determine, for the positions shown, (a) the velocity of B relative to A, (b) the acceleration of B relative to A.

SOLUTION

First note $\quad v_A = 450 \text{ km/h} \quad v_B = 540 \text{ km/h} = 150 \text{ m/s}$

(a) We have $\quad \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown.



We have $\quad v_{B/A}^2 = 450^2 + 540^2 - 2(450)(540)\cos 60°$

or $\quad v_{B/A} = 501.10 \text{ km/h}$

and $\quad \dfrac{540}{\sin\alpha} = \dfrac{501.10}{\sin 60°}$

or $\quad \alpha = 68.9°$

$\mathbf{v}_{B/A} = 501 \text{ km/h} \nearrow 68.9°$ ◀

(b) First note $\quad \mathbf{a}_A = 8 \text{ m/s}^2 \rightarrow (\mathbf{a}_B)_t = 3 \text{ m/s}^2 \searrow 60°$

Now $\quad (a_B)_n = \dfrac{v_B^2}{\rho_B} = \dfrac{(150 \text{ m/s})^2}{300 \text{ m}}$

or $\quad (\mathbf{a}_B)_n = 75 \text{ m/s}^2 \nearrow 30°$ ◀

Then $\quad \mathbf{a}_B = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n$

$$= 3(-\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j}) + 75(-\cos 30°\,\mathbf{i} - \sin 30°\,\mathbf{j})$$

$$= -(66.452 \text{ m/s}^2)\mathbf{i} - (34.902 \text{ m/s}^2)\mathbf{j}$$

Finally $\quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

or $\quad \mathbf{a}_{B/A} = (-66.452\mathbf{i} - 34.902\mathbf{j}) - (8\mathbf{i})$

$$= -(74.452 \text{ m/s}^2)\mathbf{i} - (34.902 \text{ m/s}^2)\mathbf{j}$$

or $\quad \mathbf{a}_{B/A} = 82.2 \text{ m/s}^2 \nearrow 25.1°$ ◀

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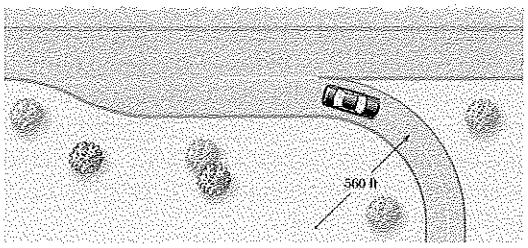
PROBLEM 11.141



A motorist traveling along a straight portion of a highway is decreasing the speed of his automobile at a constant rate before exiting from the highway onto a circular exit ramp with a radius of 560-ft. He continues to decelerate at the same constant rate so that 10 s after entering the ramp, his speed has decreased to 20 mi/h, a speed which he then maintains. Knowing that at this constant speed the total acceleration of the automobile is equal to one-quarter of its value prior to entering the ramp, determine the maximum value of the total acceleration of the automobile.

SOLUTION

First note $$v_{10} = 20 \text{ mi/h} = \frac{88}{3} \text{ ft/s}$$

While the car is on the straight portion of the highway.

$$a = a_{\text{straight}} = a_t$$

and for the circular exit ramp

$$a = \sqrt{a_t^2 + a_n^2}$$

where $$a_n = \frac{v^2}{\rho}$$

By observation, $a_{\max}$ occurs when v is maximum, which is at $t = 0$ when the car first enters the ramp.

For uniformly decelerated motion

$$v = v_0 + a_t t$$

and at $t = 10$ s: $$v = \text{constant} \Rightarrow a = a_n = \frac{v_{10}^2}{\rho}$$

$$a = \frac{1}{4} a_{\text{st.}}$$

Then $$a_{\text{straight}} = a_t \Rightarrow \frac{1}{4} a_t = \frac{v_{10}^2}{\rho} = \frac{\left(\frac{88}{3} \text{ ft/s}\right)^2}{560 \text{ ft}}$$

or $$a_t = -6.1460 \text{ ft/s}^2$$

(The car is decelerating; hence the minus sign.)

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PROBLEM 11.141 (Continued)

Then at $t = 10$ s: $$\frac{88}{3}\text{ ft/s} = v_0 + (-6.1460\text{ ft/s}^2)(10\text{ s})$$

or $$v_0 = 90.793\text{ ft/s}$$

Then at $t = 0$: $$a_{\max} = \sqrt{a_t^2 + \left(\frac{v_0^2}{\rho}\right)^2}$$

$$= \left\{(-6.1460\text{ ft/s}^2)^2 + \left[\frac{(90.793\text{ ft/s})^2}{560\text{ ft}}\right]^2\right\}^{1/2}$$

or $$a_{\max} = 15.95\text{ ft/s}^2 \blacktriangleleft$$

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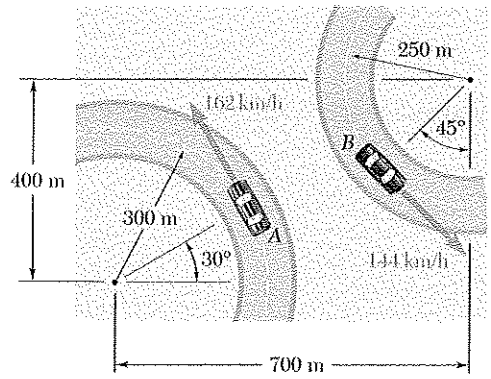
PROBLEM 11.142



Racing cars A and B are traveling on circular portions of a race track. At the instant shown, the speed of A is decreasing at the rate of 7 m/s^2, and the speed of B is increasing at the rate of 2 m/s^2. For the positions shown, determine (a) the velocity of B relative to A, (b) the acceleration of B relative to A.

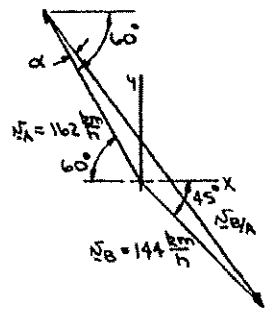
SOLUTION

First note $v_A = 162 \text{ km/h} = 45 \text{ m/s}$

$v_B = 144 \text{ km/h} = 40 \text{ m/s}$

(a) We have $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown.



We have $v_{B/A}^2 = 162^2 + 144^2 - 2(162)(144)\cos 165°$

or $v_{B/A} = 303.39 \text{ km/h}$

and $\dfrac{144}{\sin\alpha} = \dfrac{303.39}{\sin 165°}$

or $\alpha = 7.056°$

$\mathbf{v}_{B/A} = 303 \text{ km/h} \measuredangle 52.9°$ ◀

(b) First note $(\mathbf{a}_A)_t = 7 \text{ m/s}^2 \measuredangle 60°$

$(\mathbf{a}_B)_t = 2 \text{ m/s}^2 \measuredangle 45°$

Now $a_n = \dfrac{v^2}{\rho}$

Then $(a_A)_n = \dfrac{(45 \text{ m/s})^2}{300 \text{ m}}$ $\qquad (a_B)_n = \dfrac{(40 \text{ m/s})^2}{250 \text{ m}}$

or $(\mathbf{a}_A)_n = 6.75 \text{ m/s}^2 \measuredangle 30°$ $\qquad (\mathbf{a}_B)_n = 6.40 \text{ m/s}^2 \measuredangle 45°$

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PROBLEM 11.142 (Continued)

Noting that $\mathbf{a} = \mathbf{a}_t + \mathbf{a}_n$

We have
$$\mathbf{a}_A = 7(\cos 60°\mathbf{i} - \sin 60°\mathbf{j}) + 6.75(-\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$$
$$= -(2.3457 \text{ m/s}^2)\mathbf{i} - (9.4372 \text{ m/s}^2)\mathbf{j}$$

and
$$\mathbf{a}_B = 2(\cos 45°\mathbf{i} - \sin 45°\mathbf{j}) + 6.40(\cos 45°\mathbf{i} + \sin 45°\mathbf{j})$$
$$= (5.9397 \text{ m/s}^2)\mathbf{i} + (3.1113 \text{ m/s}^2)\mathbf{j}$$

Finally $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

or
$$\mathbf{a}_{B/A} = (5.9397\mathbf{i} + 3.1113\mathbf{j}) - (-2.3457\mathbf{i} - 9.4372\mathbf{j})$$
$$= (8.2854 \text{ m/s}^2)\mathbf{i} + (12.5485 \text{ m/s}^2)\mathbf{j}$$

or $\mathbf{a}_{B/A} = 15.04 \text{ m/s}^2 \angle 56.6°$ ◀

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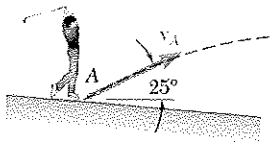
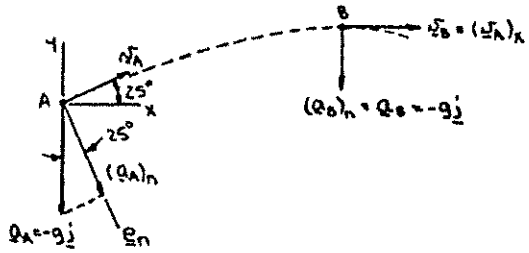
PROBLEM 11.143



A golfer hits a golfball from Point A with an initial velocity of 50 m/s at an angle of 25° with the horizontal. Determine the radius of curvature of the trajectory described by the ball (a) at Point A, (b) at the highest point of the trajectory.

SOLUTION



(a) We have $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$\rho_A = \frac{(50 \text{ m/s})^2}{(9.81 \text{ m/s}^2)\cos 25°}$$

or $$\rho_A = 281 \text{ m} \blacktriangleleft$$

(b) We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

where Point B is the highest point of the trajectory, so that

$$v_B = (v_A)_x = v_A \cos 25°$$

Then $$\rho_B = \frac{[(50 \text{ m/s}) \cos 25°]^2}{9.81 \text{ m/s}^2}$$

or $$\rho_B = 209 \text{ m} \blacktriangleleft$$

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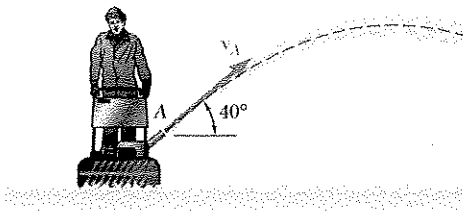
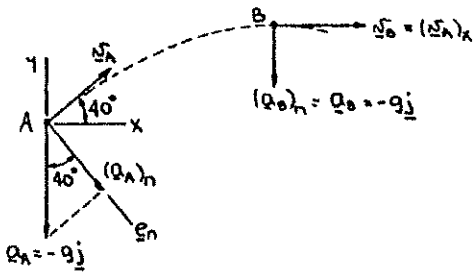


PROBLEM 11.144

From a photograph of a homeowner using a snowblower, it is determined that the radius of curvature of the trajectory of the snow was 8.5 m as the snow left the discharge chute at A. Determine (a) the discharge velocity $\mathbf{v}_A$ of the snow, (b) the radius of curvature of the trajectory at its maximum height.

SOLUTION



(a) We have $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$v_A^2 = (9.81 \cos 40°)(8.5 \text{ m})$$
$$= 63.8766 \text{ m}^2/\text{s}^2$$

or $$v_A = 7.99 \text{ m/s} \angle 40° \blacktriangleleft$$

(b) We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

where Point B is the highest point of the trajectory, so that

$$v_B = (v_A)_x = v_A \cos 40°$$

Then $$\rho_B = \frac{(63.8766 \text{ m}^2/\text{s}^2)\cos^2 40°}{9.81 \text{ m/s}^2}$$

or $$\rho_B = 3.82 \text{ m} \blacktriangleleft$$

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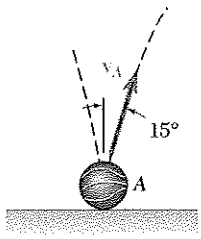
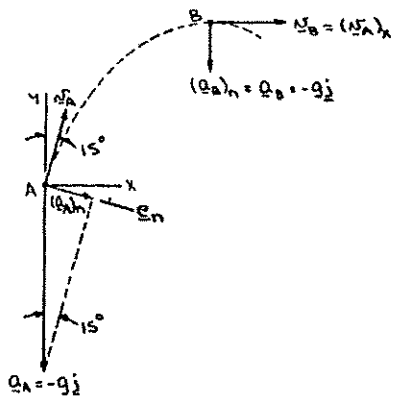


PROBLEM 11.145

A basketball is bounced on the ground at Point A and rebounds with a velocity $\mathbf{v}_A$ of magnitude 7.5 ft/s as shown. Determine the radius of curvature of the trajectory described by the ball (*a*) at Point A, (*b*) at the highest point of the trajectory.

SOLUTION



(*a*) We have $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$\rho_A = \frac{(7.5 \text{ ft/s})^2}{(32.2 \text{ ft/s}^2)\sin 15°}$$

or $$\rho_A = 6.75 \text{ ft} \blacktriangleleft$$

(*b*) We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

where Point B is the highest point of the trajectory, so that

$$v_B = (v_A)_x = v_A \sin 15°$$

Then $$\rho_B = \frac{[(7.5 \text{ ft/s}) \sin 15°]^2}{32.2 \text{ ft/s}^2}$$

or $$\rho_B = 0.1170 \text{ ft} \blacktriangleleft$$

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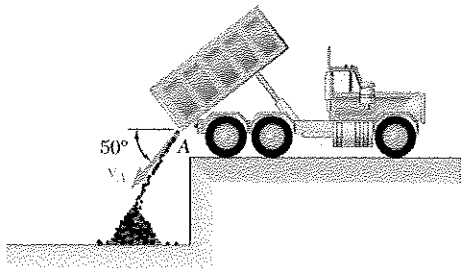
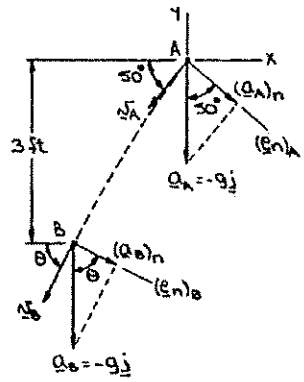


PROBLEM 11.146

Coal is discharged from the tailgate A of a dump truck with an initial velocity $\mathbf{v}_A = 6$ ft/s $\measuredangle$ 50°. Determine the radius of curvature of the trajectory described by the coal (a) at Point A, (b) at the point of the trajectory 3 ft below Point A.

SOLUTION



(a) We have $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$\rho_A = \frac{(6 \text{ ft/s})^2}{(32.2 \text{ ft/s}^2)\cos 50°}$$

or $$\rho_A = 1.739 \text{ ft}$$

(b) Horizontal motion. (Uniform)

$$(v_B)_x = (v_A)_x = (6 \text{ ft/s}) \cos 50° = 3.8567 \text{ ft/s}$$

Vertical motion. (Uniformly accelerated motion)

We have $$v_y^2 = (v_A)_y^2 - 2g\left(y - y_A^0\right) \quad (g = 32.2 \text{ ft/s}^2)$$

where $$(v_A)_y = (6 \text{ ft/s}) \sin 50° = 4.5963 \text{ ft/s}$$

At Point B, $y = -3$ ft: $$(v_B)_y^2 = (4.5963 \text{ ft/s})^2 - 2(32.2 \text{ ft/s}^2)(-3 \text{ ft})$$

or $$(v_B)_y = 14.6399 \text{ ft/s}$$

Then $$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{(3.8567)^2 + (14.6399)^2} = 15.1394 \text{ ft/s}$$

and $$\tan\theta = \frac{v_y}{v_x} = \frac{14.6399}{3.8567}$$

or $$\theta = 75.24°$$

Now $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

or $$\rho_B = \frac{(15.1394 \text{ ft/s})^2}{(32.2 \text{ ft/s}^2)\cos 75.24°}$$

or $$\rho_B = 27.9 \text{ ft} \blacktriangleleft$$

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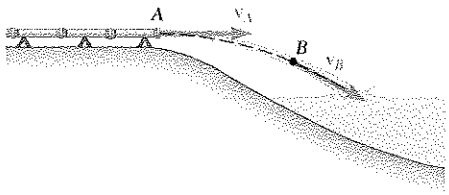
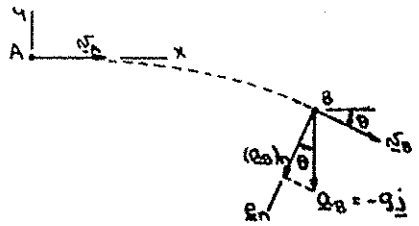


PROBLEM 11.147

A horizontal pipe discharges at Point A a stream of water into a reservoir. Express the radius of curvature of the stream at Point B in terms of the magnitudes of the velocities $\mathbf{v}_A$ and $\mathbf{v}_B$.

SOLUTION

We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$



where $$(a_B)_n = a_B \cos\theta = g\cos\theta$$

Noting that the horizontal motion is uniform, we have

$$(v_B)_x = v_A$$

where $$(v_B)_x = v_B \cos\theta$$

$$\cos\theta = \frac{v_A}{v_B}$$

Then $$\rho_B = \frac{v_B^2}{g\left(\frac{v_A}{v_B}\right)}$$

or $$\rho_B = \frac{v_B^2}{gv_A} \blacktriangleleft$$

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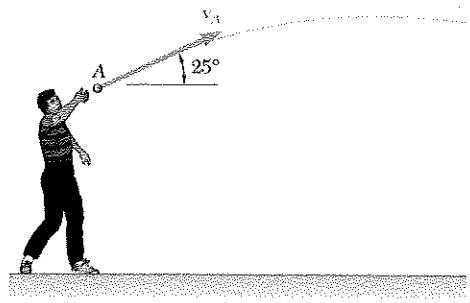

PROBLEM 11.148

A child throws a ball from Point A with an initial velocity $\mathbf{v}_A$ of 20 m/s at an angle of 25° with the horizontal. Determine the velocity of the ball at the points of the trajectory described by the ball where the radius of curvature is equal to three-quarters of its value at A.

SOLUTION

Assume that Point B and C are the points of interest, where $y_B = y_C$ and $v_B = v_C$.

Now $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$\rho_A = \frac{v_A^2}{g\cos 25°}$$

Then $$\rho_B = \frac{3}{4}\rho_A = \frac{3}{4}\frac{v_A^2}{g\cos 25°}$$

We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

where $$(a_B)_n = g\cos\theta$$

so that $$\frac{3}{4}\frac{v_A^2}{g\cos 25°} = \frac{v_B^2}{g\cos\theta}$$

or $$v_B^2 = \frac{3}{4}\frac{\cos\theta}{\cos 25°}v_A^2 \qquad (1)$$

Noting that the horizontal motion is uniform, we have

$$(v_A)_x = (v_B)_x$$

where $$(v_A)_x = v_A\cos 25° \quad (v_B)_x = v_B\cos\theta$$

Then $$v_A\cos 25° = v_B\cos\theta$$

or $$\cos\theta = \frac{v_A}{v_B}\cos 25°$$

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PROBLEM 11.148 (Continued)

Substituting for $\cos\theta$ in Eq. (1), we have

$$v_B^2 = \frac{3}{4}\left(\frac{v_A}{v_B}\cos 25°\right)\frac{v_A^2}{\cos 25°}$$

or

$$v_B^3 = \frac{3}{4}v_A^3$$

$$v_B = \sqrt[3]{\frac{3}{4}}v_A = 18.17 \text{ m/s}$$

$$\cos\theta = \sqrt[3]{\frac{4}{3}}\cos 25°$$

$$\theta = \pm 4.04°$$

$$\mathbf{v}_B = 18.17 \text{ m/s} \angle 4.04° \blacktriangleleft$$

and

$$\mathbf{v}_B = 18.17 \text{ m/s} \angle 4.04° \blacktriangleleft$$

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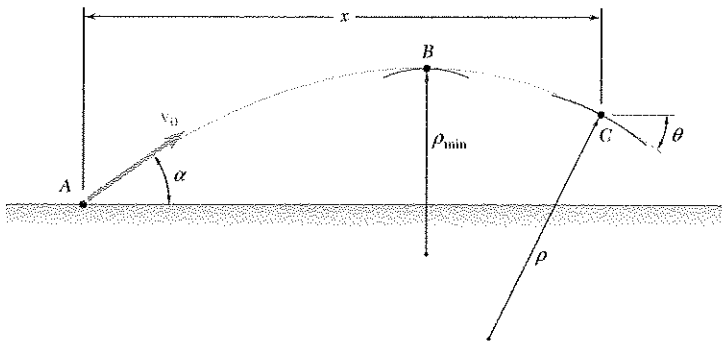


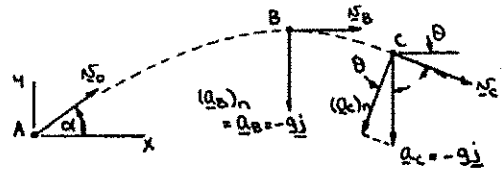
PROBLEM 11.149

A projectile is fired from Point A with an initial velocity $\mathbf{v}_0$. (a) Show that the radius of curvature of the trajectory of the projectile reaches its minimum value at the highest Point B of the trajectory. (b) Denoting by θ the angle formed by the trajectory and the horizontal at a given Point C, show that the radius of curvature of the trajectory at C is $\rho = \rho_{min}/\cos^3\theta$.

SOLUTION

For the arbitrary Point C, we have

$$(a_C)_n = \frac{v_C^2}{\rho_C}$$



or

$$\rho_C = \frac{v_C^2}{g\cos\theta}$$

Noting that the horizontal motion is uniform, we have

$$(v_A)_x = (v_C)_x$$

where $(v_A)_x = v_0\cos\alpha \quad (v_C)_x = v_C\cos\theta$

Then $v_0\cos\alpha = v_C\cos\theta$

or

$$v_C = \frac{\cos\alpha}{\cos\theta}v_0$$

so that

$$\rho_C = \frac{1}{g\cos\theta}\left(\frac{\cos\alpha}{\cos\theta}v_0\right)^2 = \frac{v_0^2\cos^2\alpha}{g\cos^3\theta}$$

(a) In the expression for ρ_C, v_0, α, and g are constants, so that ρ_C is minimum where $\cos\theta$ is maximum. By observation, this occurs at Point B where $\theta = 0$.

$$\rho_{min} = \rho_B = \frac{v_0^2\cos^2\alpha}{g} \qquad \text{Q.E.D.}$$ ◀

(b)

$$\rho_C = \frac{1}{\cos^3\theta}\left(\frac{v_0^2\cos^2\alpha}{g}\right)$$

$$\rho_C = \frac{\rho_{min}}{\cos^3\theta} \qquad \text{Q.E.D.}$$ ◀

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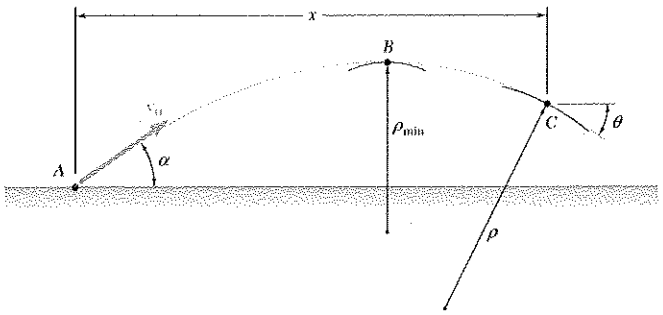
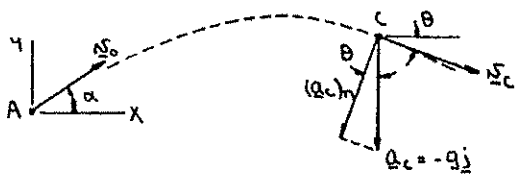


PROBLEM 11.150

A projectile is fired from Point A with an initial velocity $\mathbf{v}_0$ which forms an angle α with the horizontal. Express the radius of curvature of the trajectory of the projectile at Point C in terms of x, v_0, α, and g.

SOLUTION

We have $$(a_C)_n = \frac{v_C^2}{\rho_C}$$



or $$\rho_C = \frac{v_C^2}{g\cos\theta}$$

Noting that the horizontal motion is uniform, we have

$$(v_A)_x = (v_C)_x \qquad x = 0 + (v_0)_x t = (v_0\cos\alpha)t$$

where $(v_A)_x = v_0\cos\alpha \qquad (v_C)_x = v_C\cos\theta$

Then $$v_0\cos\alpha = v_C\cos\theta \quad \text{and} \quad (v_C)_x = v_0\cos\alpha \tag{1}$$

or $$\cos\theta = \frac{v_0}{v_C}\cos\alpha$$

so that $$\rho_C = \frac{v_C^3}{g\,v_0\cos\alpha}$$

For the uniformly accelerated vertical motion have

$$(v_C)_y = (v_0)_y - gt = v_0\sin\alpha - gt$$

From above $$x = (v_0\cos\alpha)t \quad \text{or} \quad t = \frac{x}{v_0\cos\alpha}$$

Then $$(v_C)_y = v_0\sin\alpha - g\frac{x}{v_0\cos\alpha} \tag{2}$$

Now $$v_C^2 = (v_C)_x^2 + (v_C)_y^2$$

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PROBLEM 11.150 (Continued)

Substituting for $(v_C)_x$ [Eq. (1)] and $(v_C)_y$ [Eq. (2)]

$$v_C^2 = (v_0 \cos\alpha)^2 + \left(v_0 \sin\alpha - g\frac{x}{v_0 \cos\alpha}\right)^2$$

$$= v_0^2\left(1 - \frac{2gx\tan\alpha}{v_0^2} + \frac{g^2x^2}{v_0^4\cos^2\alpha}\right)$$

or

$$v_C^3 = v_0^3\left(1 - \frac{2gx\tan\alpha}{v_0^2} + \frac{g^2x^2}{v_0^4\cos^2\alpha}\right)^{3/2}$$

Finally, substituting into the expression for ρ_C, obtain

$$\rho = \frac{v_0^2}{g\cos\alpha}\left(1 - \frac{2gx\tan\alpha}{v_0^2} + \frac{g^2x^2}{v_0^4\cos^2\alpha}\right)^{3/2}$$

◀

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PROBLEM 11.151*

Determine the radius of curvature of the path described by the particle of Problem 11.95 when $t = 0$.

SOLUTION

We have
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = R(\cos\omega_n t - \omega_n t \sin\omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$$

and
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = R\left(-\omega_n \sin\omega_n t - \omega_n \sin\omega_n t - \omega_n^2 t\cos\omega_n t\right)\mathbf{i} + R\left(\omega_n \cos\omega_n t + \omega_n \cos\omega_n t - \omega_n^2 t \sin\omega_n t\right)\mathbf{k}$$

or
$$\mathbf{a} = \omega_n R[-(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{i} + (2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{k}]$$

Now
$$v^2 = R^2(\cos\omega_n t - \omega_n t\sin\omega_n t)^2 + c^2 + R^2(\sin\omega_n t + \omega_n t\cos\omega_n t)^2$$
$$= R^2\left(1 + \omega_n^2 t^2\right) + c^2$$

Then
$$v = \left[R^2\left(1 + \omega_n^2 t^2\right) + c^2\right]^{1/2}$$

and
$$\frac{dv}{dt} = \frac{R^2\omega_n^2 t}{\left[R^2\left(1 + \omega_n^2 t^2\right) + c^2\right]^{1/2}}$$

Now
$$a^2 = a_t^2 + a_n^2$$
$$= \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$$

At $t = 0$:
$$\frac{dv}{dt} = 0$$
$$\mathbf{a} = \omega_n R(2\mathbf{k}) \quad \text{or} \quad a = 2\omega_n R$$
$$v^2 = R^2 + c^2$$

Then, with
$$\frac{dv}{dt} = 0,$$

we have
$$a = \frac{v^2}{\rho}$$

or
$$2\omega_n R = \frac{R^2 + c^2}{\rho}$$

or
$$\rho = \frac{R^2 + c^2}{2\omega_n R} \blacktriangleleft$$

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PROBLEM 11.152*

Determine the radius of curvature of the path described by the particle of Problem 11.96 when $t = 0$, $A = 3$, and $B = 1$.

SOLUTION

With $A = 3, \quad B = 1$

We have $$\mathbf{r} = (3t\cos t)\mathbf{i} + \left(3\sqrt{t^2+1}\right)\mathbf{j} + (t\sin t)\mathbf{k}$$

Now $$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 3(\cos t - t\sin t)\mathbf{i} + \left(\frac{3t}{\sqrt{t^2+1}}\right)\mathbf{j} + (\sin t + t\cos t)\mathbf{k}$$

and $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\mathbf{i} + 3\left[\frac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{t^2+1}\right]\mathbf{j}$$
$$+ (\cos t + \cos t - t\sin t)\mathbf{k}$$
$$= -3(2\sin t + t\cos t)\mathbf{i} + 3\frac{1}{(t^2+1)^{1/2}}\mathbf{j}$$
$$+ (2\cos t - t\sin t)\mathbf{k}$$

Then $$v^2 = 9(\cos t - t\sin t)^2 + 9\frac{t^2}{t^2+1} + (\sin t + t\cos t)^2$$

Expanding and simplifying yields

$$v^2 = t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3 + t)\sin 2t$$

Then $$v = [t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3 + t)\sin 2t]^{1/2}$$

and $$\frac{dv}{dt} = \frac{4t^3 + 38t + 8(-2\cos t\sin t + 4t^3\sin^2 t + 2t^4\sin t\cos t) - 8[(3t^2+1)\sin 2t + 2(t^3+t)\cos 2t]}{2[t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3+t)\sin 2t]^{1/2}}$$

Now $$a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$$

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PROBLEM 11.152* (Continued)

At $t = 0$: $\mathbf{a} = 3\mathbf{j} + 2\mathbf{k}$

or $a = \sqrt{13}\ \text{ft/s}^2$

$$\frac{dv}{dt} = 0$$

$$v^2 = 9\ (\text{ft/s})^2$$

Then, with $\frac{dv}{dt} = 0$,

we have $a = \frac{v^2}{\rho}$

or $\rho = \frac{9\ \text{ft}^2/\text{s}^2}{\sqrt{13}\ \text{ft/s}^2}$

or $\rho = 2.50\ \text{ft}$ ◀

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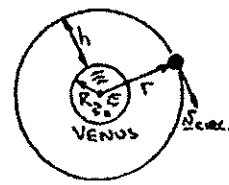
PROBLEM 11.153

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to $g(R/r)^2$, where g is the acceleration of gravity at the surface of the planet, R is the radius of the planet, and r is the distance from the center of the planet to the satellite. Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 160 km above the surface of the planet.

Venus: $g = 8.53 \text{ m/s}^2$, $R = 6161 \text{ km}$.

SOLUTION

We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$



Then $$g\frac{R^2}{r^2} = \frac{v^2}{r}$$

or $$v_{\text{circ.}} = R\sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h$$

For the given data

$$v_{\text{circ.}} = 6161 \text{ km}\sqrt{\frac{8.53 \text{ m/s}^2}{(6161+160)\times 10^3 \text{ m}}} \times \frac{3600 \text{ s}}{1 \text{ h}}$$

or $$v_{\text{circ.}} = 25.8\times 10^3 \text{ km/h} \blacktriangleleft$$

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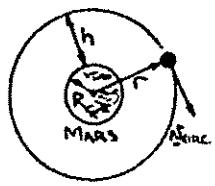
PROBLEM 11.154

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to $g(R/r)^2$, where g is the acceleration of gravity at the surface of the planet, R is the radius of the planet, and r is the distance from the center of the planet to the satellite. Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 160 km above the surface of the planet.

Mars: $g = 3.83\ \text{m/s}^2$, $R = 3332\ \text{km}$.

SOLUTION

We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$



Then $$g\frac{R^2}{r^2} = \frac{v^2}{r}$$

or $$v_{\text{circ.}} = R\sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h$$

For the given data

$$v_{\text{circ.}} = 3332\ \text{km}\sqrt{\frac{3.83\ \text{m/s}^2}{(3332+160)\times 10^3\ \text{m}}}\times\frac{3600\ \text{s}}{1\ \text{h}}$$

or $$v_{\text{circ.}} = 12.56\times 10^3\ \text{km/h} \blacktriangleleft$$

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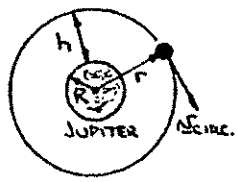
PROBLEM 11.155

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to $g(R/r)^2$, where g is the acceleration of gravity at the surface of the planet, R is the radius of the planet, and r is the distance from the center of the planet to the satellite. Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 160 km above the surface of the planet.

Jupiter: $g = 26.0\ \text{m/s}^2$, $R = 69{,}893\ \text{km}$.

SOLUTION

We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$



Then $$g\frac{R^2}{r^2} = \frac{v^2}{r}$$

or $$v_{\text{circ.}} = R\sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h$$

For the given data

$$v_{\text{circ.}} = 69{,}893\sqrt{\frac{26.0\ \text{m/s}^2}{(69{,}893+160)\times 10^3\ \text{m}}}\times\frac{3600\ \text{s}}{1\ \text{h}}$$

or $$v_{\text{circ.}} = 153.3\times 10^3\ \text{km/h} \blacktriangleleft$$

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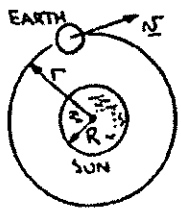
PROBLEM 11.156

Knowing that the diameter of the sun is 864,000 mi and that the acceleration of gravity at its surface is 900 ft/s^2, determine the radius of the orbit of the indicated planet around the sun assuming that the orbit is circular. (See information given in Problems 11.153–11.155.)

Earth: $(v_{\text{mean}})_{\text{orbit}} = 66,600$ mi/h

SOLUTION

We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$



Then $$g\frac{R^2}{r^2} = \frac{v^2}{r}$$

or $$r = g\left(\frac{R}{v}\right)^2 \quad \text{where} \quad R = \frac{1}{2}d$$

For the given data

$$r_{\text{earth}} = (900 \text{ ft/s}^2)\left(\frac{\frac{1}{2}\times 864,000 \text{ mi}}{66,600 \text{ mi/h}}\right)^2 \times \frac{1 \text{ mi}}{5280 \text{ ft}} = \left(\frac{3600 \text{ s}}{1 \text{ h}}\right)^2$$

or $$r_{\text{earth}} = 92.9\times 10^6 \text{ mi} \blacktriangleleft$$

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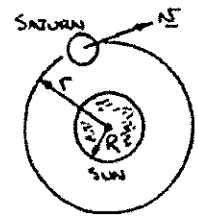
PROBLEM 11.157

Knowing that the diameter of the sun is 864,000 mi and that the acceleration of gravity at its surface is 900 ft/s^2, determine the radius of the orbit of the indicated planet around the sun assuming that the orbit is circular. (See information given in Problems 11.153–11.155.)

Saturn: $(v_{\text{mean}})_{\text{orbit}} = 21{,}580$ mi/h

SOLUTION

We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$



Then $$g\frac{R^2}{r^2} = \frac{v^2}{r}$$

or $$r = g\left(\frac{R}{v}\right)^2 \quad \text{where} \quad R = \frac{1}{2}d$$

For the given data

$$r_{\text{saturn}} = (900 \text{ ft/s}^2)\left(\frac{\frac{1}{2}\times 864{,}000 \text{ mi}}{21{,}580 \text{ mi/h}}\right)^2 \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \left(\frac{3600 \text{ s}}{1 \text{ h}}\right)^2$$

or $$r_{\text{saturn}} = 885\times 10^6 \text{ mi} \blacktriangleleft$$

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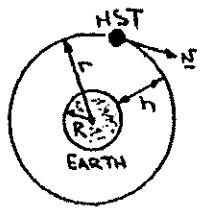
PROBLEM 11.158

Knowing that the radius of the earth is 6370 km, determine the time of one orbit of the Hubble Space Telescope, knowing that the telescope travels in a circular orbit 590 km above the surface of the earth. (See information given in Problems 11.153–11.155.)

SOLUTION

We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$



Then $$g\frac{R^2}{r^2} = \frac{v^2}{r}$$

or $$v = R\sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h$$

The circumference s of the circular orbit is equal to

$$s = 2\pi r$$

Assuming that the speed of the telescope is constant, we have

$$s = vt_{\text{orbit}}$$

Substituting for s and v

$$2\pi r = R\sqrt{\frac{g}{r}}\, t_{\text{orbit}}$$

or $$t_{\text{orbit}} = \frac{2\pi}{R}\frac{r^{3/2}}{\sqrt{g}}$$

$$= \frac{2\pi}{6370 \text{ km}} \frac{[(6370 + 590) \text{ km}]^{3/2}}{[9.81 \times 10^{-3} \text{ km/s}^2]^{1/2}} \times \frac{1 \text{ h}}{3600 \text{ s}}$$

or $$t_{\text{orbit}} = 1.606 \text{ h} \blacktriangleleft$$

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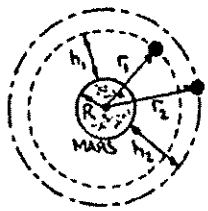
PROBLEM 11.159

A satellite is traveling in a circular orbit around Mars at an altitude of 180 mi. After the altitude of the satellite is adjusted, it is found that the time of one orbit has increased by 10 percent. Knowing that the radius of Mars is 2071 mi, determine the new altitude of the satellite. (See information given in Problems 11.153–11.155.)

SOLUTION

We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$



Then $$g\frac{R^2}{r^2} = \frac{v^2}{r}$$

or $$v = R\sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h$$

The circumference s of a circular orbit is equal to

$$s = 2\pi r$$

Assuming that the speed of the satellite in each orbit is constant, we have

$$s = vt_{\text{orbit}}$$

Substituting for s and v

$$2\pi r = R\sqrt{\frac{g}{r}}\, t_{\text{orbit}}$$

or $$t_{\text{orbit}} = \frac{2\pi}{R}\frac{r^{3/2}}{\sqrt{g}}$$

$$= \frac{2\pi}{R}\frac{(R+h)^{3/2}}{\sqrt{g}}$$

Now $$(t_{\text{orbit}})_2 = 1.1(t_{\text{orbit}})_1$$

or $$\frac{2\pi}{R}\frac{(R+h_2)^{3/2}}{\sqrt{g}} = 1.1\frac{2\pi}{R}\frac{(R+h_1)^{3/2}}{\sqrt{g}}$$

or $$h_2 = (1.1)^{2/3}(R+h_1) - R$$

$$= (1.1)^{2/3}(2071+180)\text{ mi} - (2071\text{ mi})$$

or $$h_2 = 328\text{ mi} \blacktriangleleft$$

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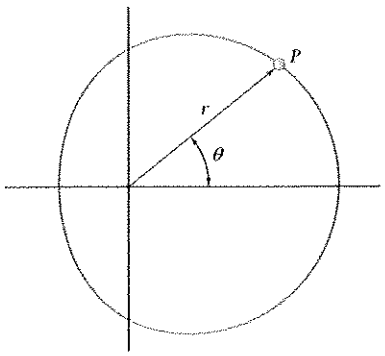

PROBLEM 11.161

The path of a particle P is a limaçon. The motion of the particle is defined by the relations $r = b(2 + \cos \pi t)$ and $\theta = \pi t$, where t and θ are expressed in seconds and radians, respectively. Determine (a) the velocity and the acceleration of the particle when $t = 2$ s, (b) the values of θ for which the magnitude of the velocity is maximum.

SOLUTION

We have $r = b(2 + \cos \pi t) \qquad \theta = \pi t$

Then $\dot{r} = -\pi b \sin \pi t \qquad \dot{\theta} = \pi$

and $\ddot{r} = -\pi^2 b \cos \pi t \qquad \ddot{\theta} = 0$

Now $\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = -(\pi b \sin \pi t)\mathbf{e}_r + \pi b(2 + \cos \pi t)\mathbf{e}_\theta$

and $\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$

$= [-\pi^2 b \cos \pi t - \pi^2 b(2 + \cos \pi t)]\mathbf{e}_r$

$+ (0 - 2\pi^2 b \sin \pi t)\mathbf{e}_\theta$

$= -2\pi^2 b[(1 + \cos \pi t)\mathbf{e}_r + (\sin \pi t)\mathbf{e}_\theta]$

(a) At $t = 2$ s: $\mathbf{v} = -(0)\mathbf{e}_r + \pi b(2 + 1)\mathbf{e}_\theta$

or $\mathbf{v} = 3\pi b\mathbf{e}_\theta$ ◀

$\mathbf{a} = -2\pi^2 b[(1 + 1)\mathbf{e}_r + (0)\mathbf{e}_\theta]$

or $\mathbf{a} = -4\pi^2 b\mathbf{e}_r$ ◀

(b) We have $v = \pi b\sqrt{(-\sin \pi t)^2 + (2 + \cos \pi t)^2}$

$= \pi b\sqrt{5 + 4\cos \pi t} \qquad \theta = \pi t$

$= \pi b\sqrt{5 + 4\cos \theta}$

By observation, $v = v_{\max}$ when $\cos \theta = 1$

or $\theta = 2n\pi, \quad n = 0, 1, 2, \ldots$ ◀

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PROBLEM 11.162

The two-dimensional motion of a particle is defined by the relation $r = 2b\cos\omega t$ and $\theta = \omega t$ where b and ω are constant. Determine (a) the velocity and acceleration of the particle at any instant, (b) the radius of curvature of its path. What conclusion can you draw regarding the path of the particle?

SOLUTION

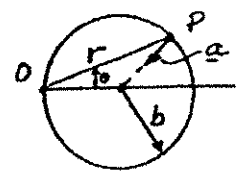
$$r = 2b\cos\omega t \qquad \theta = \omega t$$
$$\dot{r} = -2b\omega\sin\omega t \qquad \dot{\theta} = \omega$$
$$\ddot{r} = -2b\omega^2\cos\omega t \qquad \ddot{\theta} = 0$$

(a) Velocity. $\quad v_r = \dot{r} = -2b\omega\sin\omega t \quad v_\theta = r\dot{\theta} = 2b\omega\cos\omega t$

$$v^2 = v_r^2 + v_\theta^2 = (2b\omega)^2[(-\sin\omega t)^2 + (\cos\omega t)^2] = (2b\omega)^2 \qquad v = 2b\omega \blacktriangleleft$$

Acceleration.
$$a_r = \ddot{r} - r\dot{\theta}^2 = -2b\omega^2\cos\omega t - (2b\cos\omega t)(\omega)^2$$
$$= -4b\omega^2\cos\omega t$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (2b\cos\omega t)(0) + 2(-2b\omega\sin\omega t)(\omega)$$
$$= -4b\omega^2\sin\omega t$$
$$a^2 = a_r^2 + a_\theta^2 = (-4b\omega^2)^2(\cos^2\omega t + \sin^2\omega t) = (4b\omega^2)^2 \qquad a = 4b\omega^2 \blacktriangleleft$$

(b) Since $\quad v = 2b\omega = \text{constant}, \quad a_t = 0$



Thus: $\quad a_n = a = 4b\omega^2$

$$a_n = \frac{v^2}{\rho}; \quad \rho = \frac{v^2}{a_n} = \frac{(2b\omega)^2}{4b\omega^2} = b \quad \rho = b$$

For the path, $\rho = \text{constant}$.

Thus, path is a circle. ◀

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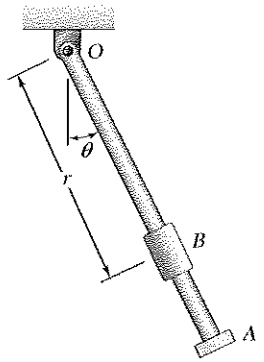


PROBLEM 11.163

The rotation of rod OA about O is defined by the relation $\theta = \pi(4t^2 - 8t)$, where θ and t are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is $r = 10 + 6\sin \pi t$, where r and t are expressed in inches and seconds, respectively. When $t = 1$ s, determine (a) the velocity of the collar, (b) the total acceleration of the collar, (c) the acceleration of the collar relative to the rod.

SOLUTION

We have $r = 10 + 6\sin \pi t$ $\theta = \pi(4t^2 - 8t)$

Then $\dot{r} = 6\pi \cos \pi t$ $\dot{\theta} = \theta\pi(t - 1)$

and $\ddot{r} = -6\pi^2 \sin \pi t$ $\ddot{\theta} = \theta\pi$

At $t = 1$ s: $r = 10$ in. $\theta = -4\pi$ rad

$\dot{r} = -6\pi$ in./s $\dot{\theta} = 0$

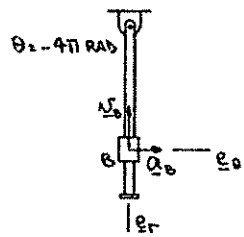
$\ddot{r} = 0$ $\ddot{\theta} = 8\pi$ rad/s^2

(a) We have $\mathbf{v}_B = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$

so that $\mathbf{v}_B = -(6\pi \text{ in./s})\mathbf{e}_r$ ◀

(b) We have $\mathbf{a}_B = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$

$= (10)(8\pi)\mathbf{e}_\theta$



or $\mathbf{a}_B = (80\pi \text{ in./s}^2)\mathbf{e}_\theta$ ◀

(c) We have $\mathbf{a}_{B/OA} = \ddot{r}$

so that $\mathbf{a}_{B/OA} = 0$ ◀

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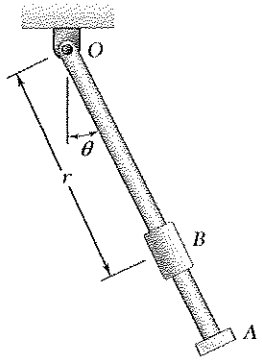


PROBLEM 11.164

The oscillation of rod OA about O is defined by the relation $\theta = (2/\pi)(\sin \pi t)$, where θ and t are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is $r = 25/(t+4)$, where r and t are expressed in inches and seconds, respectively. When $t = 1$ s, determine (a) the velocity of the collar, (b) the total acceleration of the collar, (c) the acceleration of the collar relative to the rod.

SOLUTION

We have $\quad r = \dfrac{25}{t+4} \qquad \theta = \dfrac{2}{\pi}\sin \pi t$

Then $\quad \dot{r} = -\dfrac{25}{(t+4)^2} \qquad \dot{\theta} = 2\cos \pi t$

and $\quad \ddot{r} = \dfrac{50}{(t+4)^2} \qquad \ddot{\theta} = -2\pi \sin \pi t$

At $t = 1$ s: $\quad r = 5$ in. $\qquad \theta = 0$

$\dot{r} = -1$ in./s $\qquad \dot{\theta} = -2$ rad/s

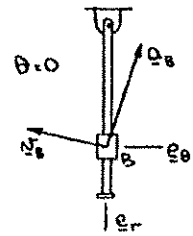
$\ddot{r} = 0.4$ in./s^2 $\qquad \ddot{\theta} = 0$

(a) We have $\quad \mathbf{v}_B = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = (-1)\mathbf{e}_r + (5)(-2)\mathbf{e}_\theta$

or $\quad \mathbf{v}_A = -(1 \text{ in./s})\mathbf{e}_r - (10 \text{ in./s})\mathbf{e}_\theta$ ◀

(b) We have $\quad \mathbf{a}_B = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\theta + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$

$= [0.4 - (5)(2)^2]\mathbf{e}_r + [0 + 2(-1)(-2)]\mathbf{e}_\theta$



or $\quad \mathbf{a}_B = -(19.6 \text{ in./s}^2)\mathbf{e}_r + (4 \text{ in./s}^2)\mathbf{e}_\theta$ ◀

(c) We have $\quad \mathbf{a}_{B/OA} = \ddot{r}$

so that $\quad \mathbf{a}_{B/OA} = (0.4 \text{ in./s}^2)\mathbf{e}_r$ ◀

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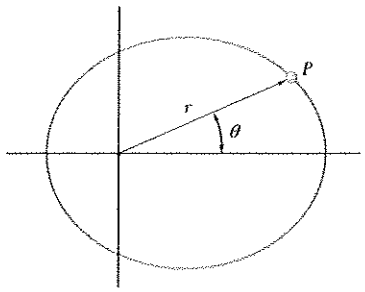

PROBLEM 11.165

The path of particle P is the ellipse defined by the relations $r = 2/(2 - \cos \pi t)$ and $\theta = \pi t$, where r is expressed in meters, t is in seconds, and θ is in radians. Determine the velocity and the acceleration of the particle when (*a*) $t = 0$, (*b*) $t = 0.5$ s.

SOLUTION

We have $r = \dfrac{2}{2 - \cos \pi t}$ $\qquad \theta = \pi t$

Then $\dot{r} = \dfrac{-2\pi \sin \pi t}{(2 - \cos \pi t)^2}$ $\qquad \dot{\theta} = \pi$

and $\ddot{r} = -2\pi \dfrac{\pi \cos \pi t(2 - \cos \pi t) - \sin \pi t(2\pi \sin \pi t)}{(2 - \cos \pi t)^3}$ $\qquad \ddot{\theta} = 0$

$$= -2\pi^2 \frac{2\cos \pi t - 1 - \sin^2 \pi t}{(2 - \cos \pi t)^3}$$

(*a*) At $t = 0$: $\qquad r = 2$ m $\qquad \theta = 0$

$\dot{r} = 0$ $\qquad \dot{\theta} = \pi$ rad/s

$\ddot{r} = -2\pi^2$ m/s^2 $\qquad \ddot{\theta} = 0$

Now $\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = (2)(\pi)\mathbf{e}_\theta$

or $\mathbf{v} = (2\pi \text{ m/s})\mathbf{e}_\theta$ ◀

and $\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$

$= [-2\pi^2 - (2)(\pi)^2]\mathbf{e}_r$

or $\mathbf{a} = -(4\pi^2 \text{ m/s}^2)\mathbf{e}_r$ ◀

(*b*) At $t = 0.5$ s: $\qquad r = 1$ m $\qquad \theta = \dfrac{\pi}{2}$ rad

$\dot{r} = \dfrac{-2\pi}{(2)^2} = -\dfrac{\pi}{2}$ m/s $\qquad \dot{\theta} = \pi$ rad/s

$\ddot{r} = -2\pi^2 \dfrac{-1-1}{(2)^3} = \dfrac{\pi^2}{2}$ m/s^2 $\qquad \ddot{\theta} = 0$

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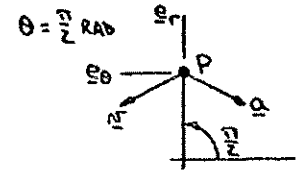
PROBLEM 11.165 (Continued)

Now $$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = \left(-\frac{\pi}{2}\right)\mathbf{e}_r + (1)(\pi)\mathbf{e}_\theta$$

or $$\mathbf{v} = -\left(\frac{\pi}{2}\text{ m/s}\right)\mathbf{e}_r + (\pi\text{ m/s})\mathbf{e}_\theta \blacktriangleleft$$

and $$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$= \left[\frac{\pi^2}{2} - (1)(\pi)^2\right]\mathbf{e}_r + \left[2\left(-\frac{\pi}{2}\right)(\pi)\right]\mathbf{e}_\theta$$



or $$\mathbf{a} = -\left(\frac{\pi^2}{2}\text{ m/s}^2\right)\mathbf{e}_r - (\pi^2\text{ m/s}^2)\mathbf{e}_\theta \blacktriangleleft$$

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PROBLEM 11.166

The two-dimensional motion of a particle is defined by the relations $r = 2a\cos\theta$ and $\theta = bt^2/2$, where a and b are constants. Determine (a) the magnitudes of the velocity and acceleration at any instant, (b) the radius of curvature of the path. What conclusion can you draw regarding the path of the particle?

SOLUTION

(a) We have $r = 2a\cos\theta$ $\qquad \theta = \frac{1}{2}bt^2$

Then $\dot{r} = -2a\theta\sin\theta$ $\qquad \dot{\theta} = bt$

and $\ddot{r} = -2a(\ddot{\theta}\sin\theta + \dot{\theta}^2 + \cos\theta)$ $\qquad \ddot{\theta} = b$

Substituting for $\dot{\theta}$ and $\ddot{\theta}$

$$\dot{r} = -2abt\sin\theta$$

$$\ddot{r} = -2ab(\sin\theta + bt^2\cos\theta)$$

Now $v_r = \dot{r} = -2abt\sin\theta$ $\qquad v_\theta = r\dot{\theta} = 2abt\cos\theta$

Then $v = \sqrt{v_r^2 + v_\theta^2} = 2abt[(-\sin\theta)^2 + (\cos\theta)^2]^{1/2}$

or $v = 2abt$ ◀

Also $a_r = \ddot{r} - r\dot{\theta}^2 = -2ab(\sin\theta + bt^2\cos\theta) - 2ab^2t^2\cos\theta$

$= -2ab(\sin\theta + 2bt^2\cos\theta)$

and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2ab\cos\theta - 4ab^2t^2\sin\theta$

$= -2ab(\cos\theta - 2bt^2\sin\theta)$

Then $a = \sqrt{a_r^2 + a_B^2} = 2ab[(\sin\theta + 2bt^2\cos\theta)^2 + (\cos\theta - 2bt^2\sin\theta)^2]^{1/2}$

or $a = 2ab\sqrt{1 + 4b^2t^4}$ ◀

(b) Now $a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$

Then $\frac{dv}{dt} = \frac{d}{dt}(2abt) = 2ab$

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PROBLEM 11.166 (Continued)

so that $$\left(2ab\sqrt{1+4b^2t^4}\right)^2 = (2ab)^2 + a_n^2$$

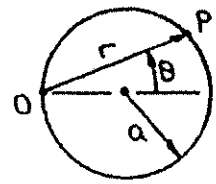
or $$4a^2b^2(1+4b^2t^4) = 4a^2b^2 + a_n^2$$

or $$a_n = 4ab^2t^2$$

Finally $$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{(2abt)^2}{4ab^2t^2}$$

or $$\rho = a \blacktriangleleft$$

Since the radius of curvature is a constant, the path is a circle of radius a. ◀



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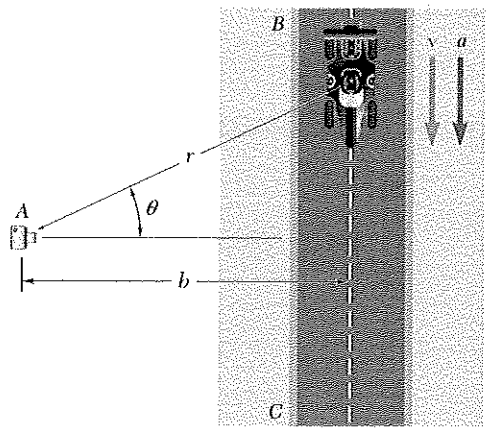
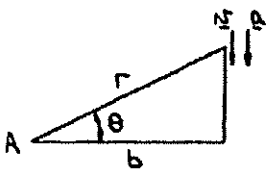


PROBLEM 11.167

To study the performance of a race car, a high-speed motion-picture camera is positioned at Point A. The camera is mounted on a mechanism which permits it to record the motion of the car as the car travels on straightway BC. Determine the speed of the car in terms of b, θ, and $\dot{\theta}$.

SOLUTION

We have $$r = \frac{b}{\cos\theta}$$



Then $$\dot{r} = \frac{b\dot{\theta}\sin\theta}{\cos^2\theta}$$

We have $$v^2 = v_r^2 + v_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$$

$$= \left(\frac{b\theta\sin\theta}{\cos^2\theta}\right)^2 + \left(\frac{b\theta}{\cos\theta}\right)^2$$

$$= \frac{b^2\dot{\theta}^2}{\cos^2\theta}\left(\frac{\sin^2\theta}{\cos^2\theta} + 1\right) = \frac{b^2\dot{\theta}^2}{\cos^4\theta}$$

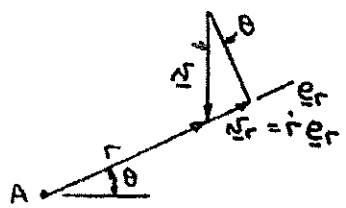
or $$v = \pm\frac{b\dot{\theta}}{\cos^2\theta}$$

For the position of the car shown, θ is decreasing; thus, the negative root is chosen.

$$v = \frac{b\dot{\theta}}{\cos^2\theta} \blacktriangleleft$$

Alternative solution.

From the diagram $$\dot{r} = -v\sin\theta$$



or $$\frac{b\dot{\theta}\sin\theta}{\cos^2\theta} = -v\sin\theta$$

or $$v = -\frac{b\dot{\theta}}{\cos^2\theta} \blacktriangleleft$$

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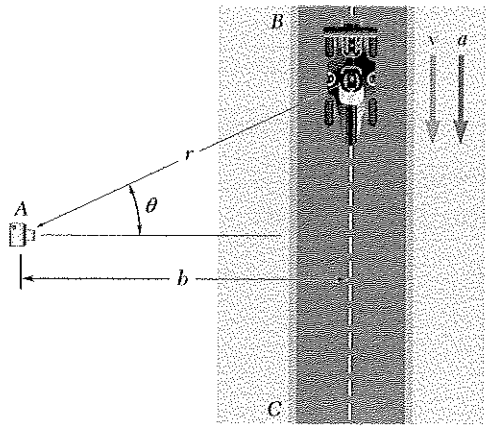
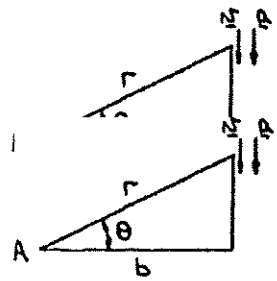


PROBLEM 11.168

Determine the magnitude of the acceleration of the race car of Problem 11.167 in terms of b, θ, $\dot{\theta}$, and $\ddot{\theta}$.

SOLUTION



We have $$r = \frac{b}{\cos\theta}$$

Then $$\dot{r} = \frac{b\dot{\theta}\sin\theta}{\cos^2\theta}$$

For rectilinear motion $$a = \frac{dv}{dt}$$

From the solution to Problem 11.167

$$v = -\frac{b\dot{\theta}}{\cos^2\theta}$$

Then $$a = \frac{d}{dt}\left(-\frac{b\dot{\theta}}{\cos^2\theta}\right)$$

$$= -b\frac{\ddot{\theta}\cos^2\theta - \dot{\theta}(-2\dot{\theta}\cos\theta\sin\theta)}{\cos^4\theta}$$

or $$a = -\frac{b}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta) \blacktriangleleft$$

Alternative solution

From above $$r = \frac{b}{\cos\theta} \qquad \dot{r} = \frac{b\dot{\theta}\sin\theta}{\cos^2\theta}$$

Then $$\ddot{r} = b\frac{(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)(\cos^2\theta) - (\dot{\theta}\sin\theta)(-2\dot{\theta}\cos\theta\sin\theta)}{\cos^4\theta}$$

$$= b\left[\frac{\ddot{\theta}\sin\theta}{\cos^2\theta} + \frac{\dot{\theta}^2(1+\sin^2\theta)}{\cos^3\theta}\right]$$

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PROBLEM 11.168 (Continued)

Now $$a^2 = a_r^2 + a_\theta^2$$

where $$a_r = \ddot{r} - r\dot{\theta}^2 = b\left[\frac{\ddot{\theta}\sin\theta}{\cos^2\theta} + \frac{\dot{\theta}^2(1+\sin^2\theta)}{\cos^2\theta}\right] - \frac{b\dot{\theta}^2}{\cos\theta}$$

$$= \frac{b}{\cos^2\theta}\left(\ddot{\theta}\sin\theta + \frac{2\dot{\theta}^2\sin^2\theta}{\cos\theta}\right)$$

$$a_r = \frac{b\sin\theta}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)$$

and $$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{b\ddot{\theta}}{\cos\theta} + 2\frac{b\dot{\theta}^2\sin\theta}{\cos^2\theta}$$

$$= \frac{b\cos\theta}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}\tan\theta)$$

Then $$a = \pm\frac{b}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)[(\sin\theta)^2 + (\cos\theta)^2]^{1/2}$$

For the position of the car shown, $\ddot{\theta}$ is negative; for a to be positive, the negative root is chosen.

$$a = -\frac{b}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta) \blacktriangleleft$$

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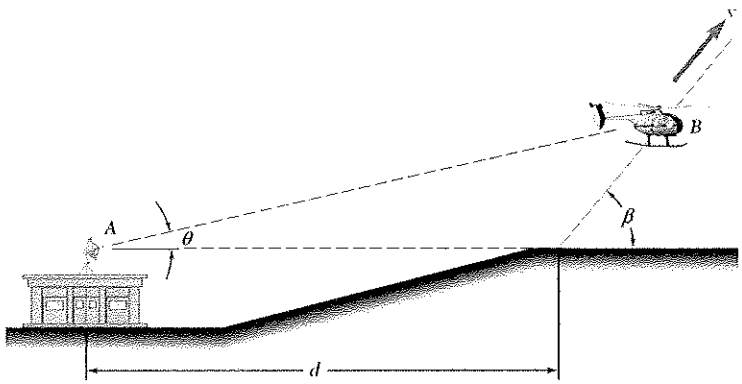

PROBLEM 11.169

After taking off, a helicopter climbs in a straight line at a constant angle β. Its flight is tracked by radar from Point A. Determine the speed of the helicopter in terms of d, β, θ, and $\dot{\theta}$.

SOLUTION

From the diagram

$$\frac{r}{\sin(180^\circ-\beta)}=\frac{d}{\sin(\beta-\theta)}$$

or $$d\sin\beta = r(\sin\beta\cos\theta-\cos\beta\sin\theta)$$

or $$r = d\frac{\tan\beta}{\tan\beta\cos\theta-\sin\theta}$$

Then $$\dot{r} = d\tan\beta\frac{-(-\tan\beta\sin\theta-\cos\theta)}{(\tan\beta\cos\theta-\sin\theta)^2}\dot{\theta}$$

$$= d\theta\tan\beta\frac{\tan\beta\sin\theta+\cos\theta}{(\tan\beta\cos\theta-\sin\theta)^2}$$

From the diagram

$$v_r = v\cos(\beta-\theta) \qquad \text{where} \qquad v_r=\dot{r}$$

Then

$$d\dot{\theta}\tan\beta\frac{\tan\beta\sin\theta+\cos\theta}{(\tan\beta\cos\theta-\sin\theta)^2} = v(\cos\beta\cos\theta+\sin\beta\sin\theta)$$

$$= v\cos\beta(\tan\beta\sin\theta+\cos\theta)$$

or $$v = \frac{d\dot{\theta}\tan\beta\sec\beta}{(\tan\beta\cos\theta-\sin\theta)^2}$$ ◀

Alternative solution.

We have $$v^2 = v_r^2+v_\theta^2 = (\dot{r})^2+(r\dot{\theta})^2$$

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PROBLEM 11.169 (Continued)

Using the expressions for r and $\dot{r}$ from above

$$v = \left[d\dot{\theta} \tan\beta \frac{\tan\beta \sin\theta + \cos\theta}{(\tan\beta \cos\theta - \sin\theta)^2} \right]^2$$

or

$$v = \pm \frac{d\dot{\theta} \tan\beta}{(\tan\beta \cos\theta - \sin\theta)} \left[\frac{(\tan\beta \sin\theta + \cos\theta)^2}{(\tan\beta \cos\theta - \sin\theta)^2} + 1 \right]^{1/2}$$

$$= \pm \frac{d\dot{\theta} \tan\beta}{(\tan\beta \cos\theta - \sin\theta)} \left[\frac{\tan^2\beta + 1}{(\tan\beta \cos\theta - \sin\theta)^2} \right]^{1/2}$$

Note that as θ increases, the helicopter moves in the indicated direction. Thus, the positive root is chosen.

$$v = \frac{d\dot{\theta} \tan\beta \sec\beta}{(\tan\beta \cos\theta - \sin\theta)^2} \blacktriangleleft$$

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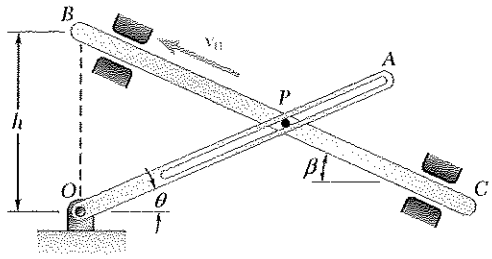
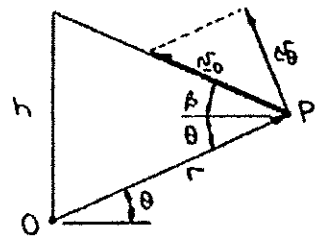


PROBLEM 11.170*

Pin P is attached to BC and slides freely in the slot of OA. Determine the rate of change $\dot{\theta}$ of the angle θ, knowing that BC moves at a constant speed v_0. Express your answer in terms of v_0, h, β, and θ.

SOLUTION

From the diagram



$$\frac{r}{\sin(90^\circ-\beta)}=\frac{h}{\sin(\beta+\theta)}$$

or $$r(\sin\beta\cos\theta+\cos\beta\sin\theta)=h\cos\beta$$

or $$r=\frac{h}{\tan\beta\cos\theta+\sin\theta}$$

Also, $$v_\theta=v_0\sin(\beta+\theta) \quad \text{where} \quad v_\theta=r\dot{\theta}$$

Then $$\frac{h\dot{\theta}}{\tan\beta\cos\theta+\sin\theta}=v_0(\sin\beta\cos\theta+\cos\beta\sin\theta)$$

or $$\dot{\theta}=\frac{v_0\cos\beta}{h}(\tan\beta\cos\theta+\sin\theta)^2 \blacktriangleleft$$

Alternative solution.

From above $$r=\frac{h}{\tan\beta\cos\theta+\sin\theta}$$

Then $$\dot{r}=h\frac{\tan\beta\sin\theta-\cos\theta}{(\tan\beta\cos\theta+\sin\theta)^2}\dot{\theta}$$

Now $$v_0^2=v_r^2+v_\theta^2=(\dot{r})^2+(r\dot{\theta})^2$$

or $$v_0^2=\left[h\dot{\theta}\frac{\tan\beta\sin\theta-\cos\theta}{(\tan\beta\cos\theta+\sin\theta)^2}\right]^2+\left(\frac{h\dot{\theta}}{\tan\beta\cos\beta+\sin\theta}\right)^2$$

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PROBLEM 11.170* (Continued)

or

$$v_0 = \pm \frac{h\dot{\theta}}{\tan\beta\cos\theta + \sin\theta} + \left[\frac{(\tan\beta\sin\theta - \cos\theta)^2}{(\tan\beta\cos\theta + \sin\theta)^2} + 1\right]^{1/2}$$

$$= \pm \frac{h\dot{\theta}}{\tan\beta\cos\theta + \sin\theta}\left[\frac{\tan^2\beta + 1}{(\tan\beta\cos\theta + \sin\theta)^2}\right]^{1/2}$$

Note that as θ increases, member BC moves in the indicated direction. Thus, the positive root is chosen.

$$\dot{\theta} = \frac{v_0\cos\beta}{h}(\tan\beta\cos\theta + \sin\theta)^2 \blacktriangleleft$$

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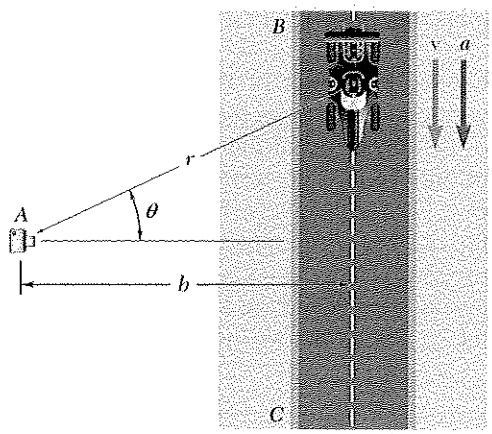
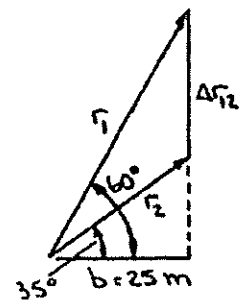


PROBLEM 11.171

For the race car of Problem 11.167, it was found that it took 0.5 s for the car to travel from the position $\theta = 60°$ to the position $\theta = 35°$. Knowing that $b = 25$ m, determine the average speed of the car during the 0.5-s interval.

SOLUTION

From the diagram:



$$\Delta r_{12} = 25 \tan 60° - 25 \tan 35°$$
$$= 25.796 \text{ m}$$

Now
$$v_{\text{ave}} = \frac{\Delta r_{12}}{\Delta t_{12}}$$
$$= \frac{25.796 \text{ m}}{0.5 \text{ s}}$$
$$= 51.592 \text{ m/s}$$

or
$$v_{\text{ave}} = 185.7 \text{ km/h} \blacktriangleleft$$

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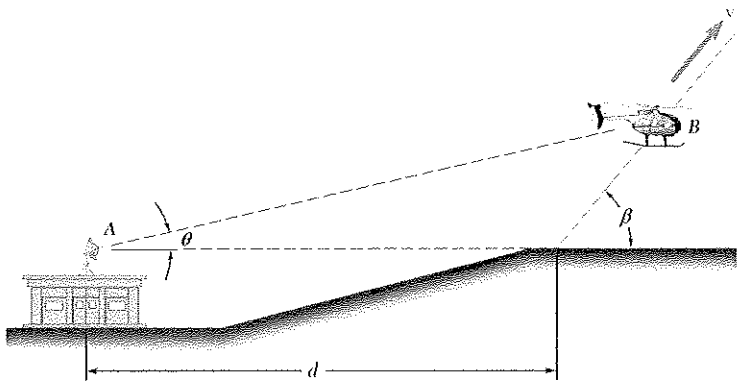


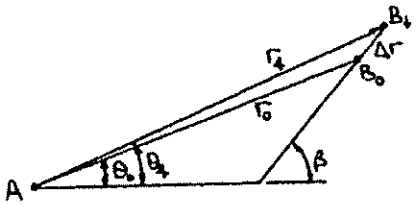
PROBLEM 11.172

For the helicopter of Problem 11.169, it was found that when the helicopter was at B, the distance and the angle of elevation of the helicopter were $r = 3000$ ft and $\theta = 20°$, respectively. Four seconds later, the radar station sighted the helicopter at $r = 3320$ ft and $\theta = 23.1°$. Determine the average speed and the angle of climb β of the helicopter during the 4-s interval.

SOLUTION

We have $r_0 = 3000$ ft $\quad \theta_0 = 20°$

$r_4 = 3320$ ft $\quad \theta_4 = 23.1°$



From the diagram:

$$\Delta r^2 = 3000^2 + 3320^2 - 2(3000)(3320)\cos(23.1° - 20°)$$

or $\Delta r = 362.70$ ft

Now $$v_{\text{ave}} = \frac{\Delta r}{\Delta t} = \frac{362.70 \text{ ft}}{4 \text{ s}} = 90.675 \text{ ft/s}$$

or $v_{\text{ave}} = 61.8$ mi/h ◀

Also, $\Delta r \cos\beta = r_4 \cos\theta_4 - r_0 \cos\theta_0$

or $$\cos\beta = \frac{3320\cos 23.1° - 3000\cos 20°}{362.70}$$

or $\beta = 49.7°$ ◀

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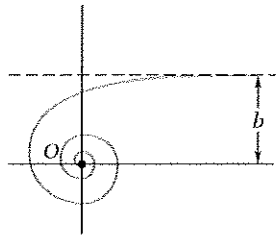

Hyperbolic spiral $r\theta = b$

PROBLEM 11.173

A particle moves along the spiral shown; determine the magnitude of the velocity of the particle in terms of b, θ, and $\dot{\theta}$.

SOLUTION

Hyperbolic spiral.

$$r = \frac{b}{\theta}$$

$$\dot{r} = \frac{dr}{dt} = -\frac{b}{\theta^2}\frac{d\theta}{dt} = -\frac{b}{\theta^2}\dot{\theta}$$

$$v_r = \dot{r} = -\frac{b}{\theta^2}\dot{\theta} \qquad v_\theta = r\dot{\theta} = \frac{b}{\theta}\dot{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = b\dot{\theta}\sqrt{\left(-\frac{1}{\theta^2}\right)^2 + \left(\frac{1}{\theta}\right)^2}$$

$$= \frac{b\dot{\theta}}{\theta^2}\sqrt{1+\theta^2}$$

$$v = \frac{b}{\theta^2}\sqrt{1+\theta^2}\,\dot{\theta} \blacktriangleleft$$

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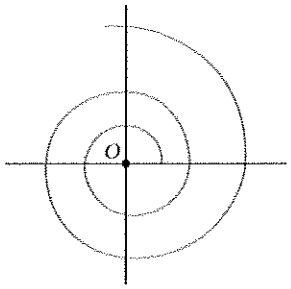

Logarithmic spiral $r = e^{b\theta}$

PROBLEM 11.174

A particle moves along the spiral shown; determine the magnitude of the velocity of the particle in terms of b, θ, and $\dot{\theta}$.

SOLUTION

Logarithmic spiral.

$$r = e^{b\theta}$$

$$\dot{r} = \frac{dr}{dt} = be^{b\theta}\frac{d\theta}{dt} = be^{b\theta}\dot{\theta}$$

$$v_r = \dot{r} = be^{b\theta}\dot{\theta} \quad v_\theta = r\dot{\theta} = e^{b\theta}\dot{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = e^{b\theta}\dot{\theta}\sqrt{b^2 + 1}$$

$$v = e^{b\theta}\sqrt{1 + b^2}\ \dot{\theta} \blacktriangleleft$$

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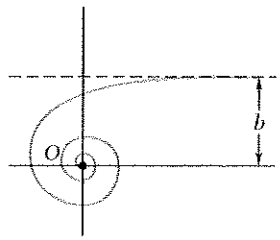

Hyperbolic spiral $r\theta = b$

PROBLEM 11.175

A particle moves along the spiral shown. Knowing that $\dot{\theta}$ is constant and denoting this constant by ω, determine the magnitude of the acceleration of the particle in terms of b, θ, and ω.

SOLUTION

Hyperbolic spiral. $r = \dfrac{b}{\theta}$

From Problem 11.173 $\dot{r} = -\dfrac{b}{\theta^2}\dot{\theta}$

$$\ddot{r} = -\frac{b}{\theta^2}\ddot{\theta} + \frac{2b}{\theta^3}\dot{\theta}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{b}{\theta^2}\ddot{\theta} + \frac{2b}{\theta^3}\dot{\theta}^2 - \frac{b}{\theta}\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{b}{\theta}\ddot{\theta} + 2\left(-\frac{b}{\theta^2}\dot{\theta}\right)\dot{\theta} = \frac{b}{\theta}\ddot{\theta} - 2\frac{b}{\theta^2}\dot{\theta}^2$$

Since $\dot{\theta} = \omega = \text{constant}$, $\ddot{\theta} = 0$, and we write:

$$a_r = +\frac{2b}{\theta^3}\omega^2 - \frac{b}{\theta}\omega^2 = \frac{b\omega^2}{\theta^3}(2 - \theta^2)$$

$$a_\theta = -2\frac{b}{\theta^2}\omega^2 = -\frac{b\omega^2}{\theta^3}(2\theta)$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \frac{b\omega^2}{\theta^3}\sqrt{(2-\theta^2)^2 + (2\theta)^2} = \frac{b\omega^2}{\theta^3}\sqrt{4 - 4\theta^2 + \theta^4 + 4\theta^2}$$

$$a = \frac{b\omega^2}{\theta^3}\sqrt{4 + \theta^4} \blacktriangleleft$$

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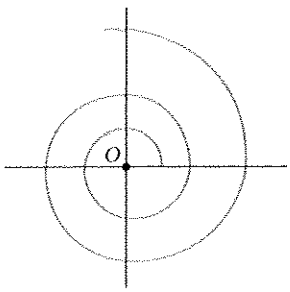

Logarithmic spiral $r = e^{b\theta}$

PROBLEM 11.176

A particle moves along the spiral shown. Knowing that $\dot{\theta}$ is constant and denoting this constant by ω, determine the magnitude of the acceleration of the particle in terms of b, θ, and ω.

SOLUTION

Logarithmic spiral. $r = e^{b\theta}$

$$\dot{r} = \frac{dr}{dt} = be^{b\theta}\dot{\theta}$$

$$\ddot{r} = be^{b\theta}\ddot{\theta} + b^2e^{b\theta}\dot{\theta}^2 = be^{b\theta}(\ddot{\theta} + b\dot{\theta}^2)$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = be^{b\theta}(\ddot{\theta} + b\dot{\theta}^2) - e^{b\theta}\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = e^{b\theta}\ddot{\theta} + 2(be^{b\theta}\dot{\theta})\dot{\theta}$$

Since $\dot{\theta} = \omega = \text{constant}$, $\ddot{\theta} = \theta$, and we write

$$a_r = be^{b\theta}(b\omega^2) - e^{b\theta}\omega^2 = e^{b\theta}(b^2 - 1)\omega^2$$

$$a_\theta = 2be^{b\theta}w^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = e^{b\theta}\omega^2\sqrt{(b^2-1)^2 + (2b)^2}$$

$$= e^{b\theta}\omega^2\sqrt{b^4 - 2b^2 + 1 + 4b^2} = e^{b\theta}\omega^2\sqrt{b^4 + 2b^2 + 1}$$

$$= e^{b\theta}\omega^2\sqrt{(b^2+1)^2} = e^{b\theta}\omega^2(b^2+1)$$

$$a = (1 + b^2)\omega^2 e^{b\theta} \blacktriangleleft$$

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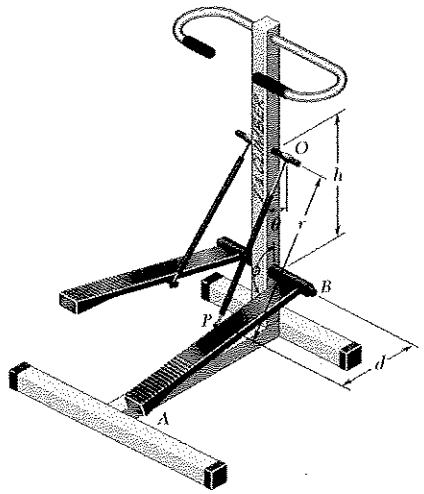


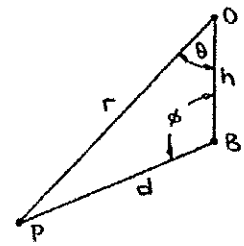
PROBLEM 11.177

Show that $\dot{r} = h\dot{\phi}\sin\theta$ knowing that at the instant shown, step AB of the step exerciser is rotating counterclockwise at a constant rate $\dot{\phi}$.

SOLUTION

From the diagram

$$r^2 = d^2 + h^2 - 2dh\cos\phi$$



Then $$2r\dot{r} = 2dh\dot{\phi}\sin\phi$$

Now $$\frac{r}{\sin\phi} = \frac{d}{\sin\theta}$$

or $$r = \frac{d\sin\phi}{\sin\theta}$$

Substituting for r in the expression for $\dot{r}$

$$\left(\frac{d\sin\phi}{\sin\theta}\right)\dot{r} = dh\dot{\phi}\sin\phi$$

or $$\dot{r} = h\dot{\phi}\sin\theta \qquad \text{Q.E.D.}$$ ◀

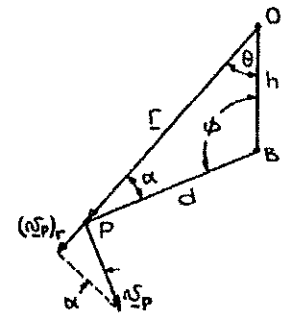
Alternative solution.

First note $$\alpha = 180° - (\phi + \theta)$$

Now $$\mathbf{v} = \mathbf{v}_r + \mathbf{v}_\theta = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

With B as the origin

$$v_P = d\dot{\phi} \qquad (d = \text{constant} \Rightarrow \dot{d} = 0)$$



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PROBLEM 11.177 (Continued)

With O as the origin $\quad (v_P)_r = \dot{r}$

where $\quad (v_P)_r = v_P \sin\alpha$

Then $\quad \dot{r} = d\dot{\phi}\sin\alpha$

Now $\quad \dfrac{h}{\sin\alpha} = \dfrac{d}{\sin\theta}$

or $\quad d\sin\alpha = h\sin\theta$

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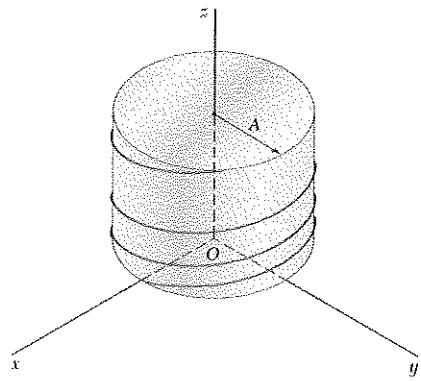

PROBLEM 11.178

The motion of a particle on the surface of a right circular cylinder is defined by the relations $R = A$, $\theta = 2\pi t$, and $z = At^2/4$, where A is a constant. Determine the magnitudes of the velocity and acceleration of the particle at any time t.

SOLUTION

We have $\quad R = A \qquad \theta = 2\pi t \qquad z = \dfrac{1}{4}At^2$

Then $\quad \dot{R} = 0 \qquad \dot{\theta} = 2\pi \qquad \dot{z} = \dfrac{1}{2}At$

and $\quad \ddot{R} = 0 \qquad \ddot{\theta} = 0 \qquad \ddot{z} = \dfrac{1}{2}A$

Now
$$\begin{aligned} v^2 &= v_R^2 + v_\theta^2 + v_z^2 \\ &= (\dot{\vec{R}}^0)^2 + (R\dot{\theta})^2 + (\dot{z})^2 \\ &= 0 + (A + 2\pi)^2 + \left(\frac{1}{2}At\right)^2 \\ &= A^2\left(4\pi^2 + \frac{1}{4}t^2\right) \end{aligned}$$

or
$$v = \frac{1}{2}A\sqrt{16\pi^2 + t^2} \blacktriangleleft$$

and
$$\begin{aligned} a^2 &= a_R^2 + a_\theta^2 + a_z^2 \\ &= (\ddot{\vec{R}}^0 - R\dot{\theta}^2)^2 + (R\ddot{\theta}^0 + 2\dot{\vec{R}}^0\theta)^2 + (\ddot{z})^2 \\ &= [-A(2\pi)^2]^2 + 0 + \left(\frac{1}{2}A\right)^2 \\ &= A^2\left(16\pi^4 + \frac{1}{4}\right) \end{aligned}$$

or
$$a = \frac{1}{2}A\sqrt{64\pi^4 + 1} \blacktriangleleft$$

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PROBLEM 11.179

The three-dimensional motion of a particle is defined by the cylindrical coordinates (see Figure 11.26) $R = A/(t+1)$, $\theta = Bt$, and $z = Ct/(t+1)$. Determine the magnitudes of the velocity and acceleration when (*a*) $t = 0$, (*b*) $t = \infty$.

SOLUTION

We have $$R = \frac{A}{t+1}, \qquad \theta = Bt, \qquad z = \frac{Ct}{t+1}$$

Then $$\dot{R} = -\frac{A}{(t+1)^2}, \qquad \dot{\theta} = B \qquad \dot{z} = C\frac{(t+1)-t}{(t+1)^2}$$

$$= \frac{C}{(t+1)^2}$$

and $$\ddot{R} = \frac{2A}{(t+1)^3} \qquad \ddot{\theta} = 0 \qquad \ddot{z} = -\frac{zC}{(t+1)^3}$$

Now $$v^2 = (v_R)^2 + (v_\theta)^2 + (v_z)^2$$

$$= (\dot{R})^2 + (R\dot{\theta})^2 + (\dot{z})^2$$

and $$a^2 = (a_R)^2 + (a_\theta)^2 + (a_z)^2$$

$$= (\ddot{R} - R\dot{\theta}^2)^2 + (R\ddot{\theta}^{0} + 2\dot{R}\dot{\theta})^2 + (\ddot{z})^2$$

(*a*) At $t = 0$: $$R = A$$

$$R = -A \qquad \dot{\theta} = B \qquad \dot{z} = C$$

$$\ddot{R} = 2A \qquad \ddot{z} = -2C$$

Then $$v^2 = (-A)^2 + (AB)^2 + (C)^2$$

or $$v = \sqrt{(1+B^2)A^2 + C^2}$$ ◀

and $$a^2 = (2A - AB^2)^2 + [z(-A)(B)]^2 + (-2C)^2$$

$$= 4A^2\left[\left(1 - \frac{1}{2}B^2\right)^2 + B^2 + \frac{C^2}{A^2}\right]$$

$$= 4\left[\left(1 + \frac{1}{4}B^4\right)A^2 + C^2\right]$$

or $$a = 2\sqrt{\left(1 + \frac{1}{4}B^4\right)A^2 + C^2}$$ ◀

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PROBLEM 11.179 (Continued)

(*b*) As $t \to \infty$: $R = 0$

$\dot{R} = 0 \quad \dot{\theta} = B \quad \dot{z} = 0$

$\ddot{R} = 0 \quad \ddot{z} = 0$

$v = 0$ ◀

and

$a = 0$ ◀

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PROBLEM 11.180*

For the conic helix of Problem 11.95, determine the angle that the osculating plane forms with the y axis.

SOLUTION

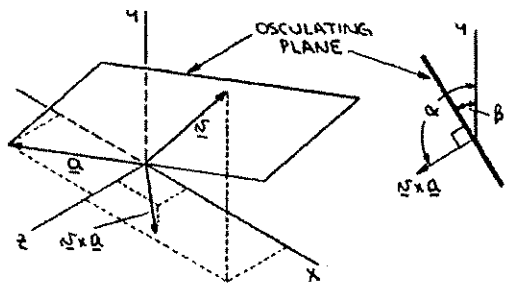
First note that the vectors **v** and **a** lie in the osculating plane.

Now
$$\mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$$

Then
$$\mathbf{v} = \frac{dr}{dt} = R(\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$$

and
$$\mathbf{a} = \frac{dv}{dt}$$
$$= R\left(-\omega_n\sin\omega_n t - \omega_n\sin\omega_n t - \omega_n^2 t\cos\omega_n t\right)\mathbf{i}$$
$$+ R\left(\omega_n\cos\omega_n t + \omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t\right)\mathbf{k}$$
$$= \omega_n R[-(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{i} + (2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{k}]$$

It then follows that the vector $(\mathbf{v}\times\mathbf{a})$ is perpendicular to the osculating plane.



$$(\mathbf{v}\times\mathbf{a}) = \omega_n R\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ R(\cos\omega_n t - \omega_n t\sin\omega_n t) & c & R(\sin\omega_n t + \omega_n t\cos\omega_n t) \\ -(2\sin\omega_n t + \omega_n t\cos\omega_n t) & 0 & (2\cos\omega_n t - \omega_n t\sin\omega_n t) \end{vmatrix}$$

$$= \omega_n R\{c(2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + R[-(\sin\omega_n t + \omega_n t\cos\omega_n t)(2\sin\omega_n t + \omega_n t\cos\omega_n t)$$
$$-(\cos\omega_n t - \omega_n t\sin\omega_n t)(2\cos\omega_n t - \omega_n t\sin\omega_n t)]\mathbf{j} + c(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$$
$$= \omega_n R\left[c(2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} - R\left(2 + \omega_n^2 t^2\right)\mathbf{j} + c(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}\right]$$

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PROBLEM 11.180* (Continued)

The angle α formed by the vector $(\mathbf{v}\times\mathbf{a})$ and the y axis is found from

$$\cos\alpha = \frac{(\mathbf{v}\times\mathbf{a})\cdot\mathbf{j}}{|(\mathbf{v}\times\mathbf{a})||\mathbf{j}|}$$

Where $$|\mathbf{j}| = 1$$

$$(\mathbf{v}\times\mathbf{a})\cdot\mathbf{j} = -\omega_n R^2\left(2+\omega_n^2 t^2\right)$$

$$|(\mathbf{v}\times\mathbf{a})| = \omega_n R\Big[c^2(2\cos\omega_n t - \omega_n t\sin\omega_n t)^2 + R^2\left(2+\omega_n^2 t^2\right)^2 + c^2(2\sin\omega_n t + \omega_n t\cos\omega_n t)^2\Big]^{1/2}$$

$$= \omega_n R\Big[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\Big]^{1/2}$$

Then $$\cos\alpha = \frac{-\omega_n R^2\left(2+\omega_n^2 t^2\right)}{\omega_n R\Big[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\Big]^{1/2}}$$

$$= \frac{-R\left(2+\omega_n^2 t^2\right)}{\Big[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\Big]^{1/2}}$$

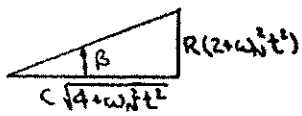
The angle β that the osculating plane forms with y axis (see the above diagram) is equal to

$$\beta = \alpha - 90^\circ$$

Then $$\cos\alpha = \cos(\beta+90^\circ) = -\sin\beta$$

$$-\sin\beta = \frac{-R\left(2+\omega_n^2 t^2\right)}{\Big[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\Big]^{1/2}}$$

Then $$\tan\beta = \frac{R\left(2+\omega_n^2 t^2\right)}{c\sqrt{4+\omega_n^2 t^2}}$$



or $$\beta = \tan^{-1}\left[\frac{R\left(2+\omega_n^2 t^2\right)}{c\sqrt{4+\omega_n^2 t^2}}\right] \blacktriangleleft$$

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PROBLEM 11.181*

Determine the direction of the binormal of the path described by the particle of Problem 11.96 when (*a*) $t = 0$, (*b*) $t = \pi/2$ s.

SOLUTION

Given: $\mathbf{r} = (At\cos t)\mathbf{i} + \left(A\sqrt{t^2+1}\right)\mathbf{j} + (Bt\sin t)\mathbf{k}$

$r - \text{ft}, \quad t - 5; \quad A = 3, \quad B - 1$

First note that $\mathbf{e}_b$ is given by

$$\mathbf{e}_b = \frac{\mathbf{v}\times\mathbf{a}}{|\mathbf{v}\times\mathbf{a}|}$$

Now $\mathbf{r} = (3t\cos t)\mathbf{i} + \left(3\sqrt{t^2+1}\right)\mathbf{j} + (t\sin t)\mathbf{k}$

Then $\mathbf{v} = \dfrac{d\mathbf{r}}{dt}$

$$= 3(\cos t - t\sin t)\mathbf{i} + \frac{3t}{\sqrt{t^2+1}}\mathbf{j} + (\sin t + t\cos t)\mathbf{k}$$

and $\mathbf{a} = \dfrac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\mathbf{i} + 3\dfrac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{t^2+1}\mathbf{j}$

$$+ (\cos t + \cos t - t\sin t)\mathbf{k}$$

$$= -3(2\sin t + t\cos t)\mathbf{i} + \frac{3}{(t^2+1)^{3/2}}\mathbf{j} + (2\cos t - t\sin t)\mathbf{k}$$

(*a*) At $t = 0$: $\mathbf{v} = (3\ \text{ft/s})\mathbf{i}$

$$\mathbf{a} = (3\ \text{ft/s}^2)\mathbf{j} + (2\ \text{ft/s}^2)\mathbf{k}$$

Then $\mathbf{v}\times\mathbf{a} = 3\mathbf{i}\times(3\mathbf{j} + 2\mathbf{k})$

$$= 3(-2\mathbf{j} + 3\mathbf{k})$$

and $|\mathbf{v}\times\mathbf{a}| = 3\sqrt{(-2)^2 + (3)^2} = 3\sqrt{13}$

Then $\mathbf{e}_b = \dfrac{3(-2\mathbf{j} + 3\mathbf{k})}{3\sqrt{13}} = \dfrac{1}{\sqrt{13}}(-2\mathbf{j} + 3\mathbf{k})$

$$\cos\theta_x = 0 \qquad \cos\theta_y = -\frac{2}{\sqrt{13}} \qquad \cos\theta_z = \frac{3}{\sqrt{13}}$$

or $\theta_x = 90° \qquad \theta_y = 123.7° \qquad \theta_z = 33.7°$ ◀

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PROBLEM 11.181* (Continued)

(*b*) At $t = \dfrac{\pi}{2}$ s:

$$\mathbf{v} = -\left(\frac{3\pi}{2}\ \text{ft/s}\right)\mathbf{i} + \left(\frac{3\pi}{\sqrt{\pi^2+4}}\ \text{ft/s}\right)\mathbf{j} + (1\ \text{ft/s})\mathbf{k}$$

$$\mathbf{a} = -(6\ \text{ft/s}^2)\mathbf{i} + \left[\frac{24}{(\pi^2+4)^{3/2}}\ \text{ft/s}^2\right]\mathbf{j} - \left(\frac{\pi}{2}\ \text{ft/s}^2\right)\mathbf{k}$$

Then

$$\mathbf{v}\times\mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\dfrac{3\pi}{2} & \dfrac{3\pi}{(\pi^2+4)^{1/2}} & 1 \\ -6 & \dfrac{24}{(\pi^2+4)^{3/2}} & -\dfrac{\pi}{2} \end{vmatrix}$$

$$= -\left[\frac{3\pi^2}{2(\pi^2+4)^{1/2}} + \frac{24}{(\pi^2+4)^{3/2}}\right]\mathbf{i} - \left(6 + \frac{3\pi^2}{4}\right)\mathbf{j} + \left[-\frac{36\pi}{(\pi^2+4)^{3/2}} + \frac{18\pi}{(\pi^2+4)^{1/2}}\right]\mathbf{k}$$

$$= -4.43984\mathbf{i} - 13.40220 + 12.99459\mathbf{k}$$

and

$$|\mathbf{v}\times\mathbf{a}| = [(-4.43984)^2 + (-13.40220)^2 + (12.99459)^2]^{1/2}$$

$$= 19.18829$$

Then

$$\mathbf{e}_b = \frac{1}{19.1829}(-4.43984\mathbf{i} - 13.40220\mathbf{j} + 12.99459\mathbf{k})$$

$$\cos\theta_x = -\frac{4.43984}{19.18829} \qquad \cos\theta_y = -\frac{13.40220}{19.18829} \qquad \cos\theta_z = \frac{12.99459}{19.18829}$$

or

$$\theta_x = 103.4^\circ \qquad \theta_y = 134.3^\circ \qquad \theta_z = 47.4^\circ \quad \blacktriangleleft$$

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PROBLEM 11.182

The motion of a particle is defined by the relation $x = 2t^3 - 15t^2 + 24t + 4$, where x and t are expressed in meters and seconds, respectively. Determine (a) when the velocity is zero, (b) the position and the total distance traveled when the acceleration is zero.

SOLUTION

$$x = 2t^3 - 15t^2 + 24t + 4$$

so
$$v = \frac{dx}{dt} = 6t^2 - 30t + 24$$

$$a = \frac{dv}{dt} = 12t - 30$$

(a) Times when $v = 0$. $\quad 0 = 6t^2 - 30t + 24 = 6\,(t^2 - 5t + 4)$

$$(t-4)(t-1) = 0 \qquad t = 1.00 \text{ s}, \quad t = 4.00 \text{ s}$$ ◀

(b) Position and distance traveled when $a = 0$.

$$a = 12t - 30 = 0 \qquad t = 2.5 \text{ s}$$

so $\quad x_2 = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4$

Final position $\quad x = 1.50 \text{ m}$ ◀

For $\quad 0 < t < 1 \text{ s}, \quad v > 0.$

For $\quad 1 \text{ s} < t < 2.5 \text{ s}, \quad v < 0.$

At $\quad t = 0, \quad x_0 = 4 \text{ m}.$

At $\quad t = 1 \text{ s}, \quad x_1 = (2)(1)^3 - (15)(1)^2 + (24)(1) + 4 = 15 \text{ m}$

Distance traveled over interval: $x_1 - x_0 = 11 \text{ m}$

For $\quad 1 \text{ s} < t < 2.5 \text{ s}, \qquad v < 0$

Distance traveled over interval

$$|x_2 - x_1| = |1.5 - 15| = 13.5 \text{ m}$$

Total distance: $\quad d = 11 + 13.5 \qquad d = 24.5 \text{ m}$ ◀

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PROBLEM 11.183

The acceleration of a particle is defined by the relation $a = -60x^{-1.5}$, where a and x are expressed in m/s^2 and meters, respectively. Knowing that the particle starts with no initial velocity at $x = 4$ m, determine the velocity of the particle when (*a*) $x = 2$ m, (*b*) $x = 1$ m, (*c*) $x = 100$ mm.

SOLUTION

We have $$v\frac{dv}{dx} = a = -60x^{-1.5}$$

When $x = 4$ m, $v = 0$: $$\int_0^v v\,dv = \int_4^x (-60x^{-1.5})dx$$

or $$\frac{1}{2}v^2 = 120[x^{-0.5}]_4^x$$

or $$v^2 = 240\left(\frac{1}{\sqrt{x}} - \frac{1}{2}\right)$$

(*a*) When $x = 2$ m: $$v^2 = 240\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)$$

or $v = -7.05$ m/s ◀

(*b*) When $x = 1$ m: $$v^2 = 240\left(1 - \frac{1}{2}\right)$$

or $v = -10.95$ m/s ◀

(*c*) When $x = 0.1$ m: $$v^2 = 240\left(\frac{1}{\sqrt{0.1}} - \frac{1}{2}\right)$$

or $v = -25.3$ m/s ◀

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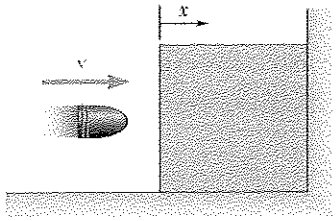

PROBLEM 11.184

A projectile enters a resisting medium at $x = 0$ with an initial velocity $v_0 = 900$ ft/s and travels 4 in. before coming to rest. Assuming that the velocity of the projectile is defined by the relation $v = v_0 - kx$, where v is expressed in ft/s and x is in feet, determine (*a*) the initial acceleration of the projectile, (*b*) the time required for the projectile to penetrate 3.9 in. into the resisting medium.

SOLUTION

First note

When $x = \frac{4}{12}$ ft, $v = 0$: $\quad 0 = (900 \text{ ft/s}) - k\left(\frac{4}{12} \text{ ft}\right)$

or $\quad k = 2700 \frac{1}{\text{s}}$

(*a*) We have $\quad v = v_0 - kx$

Then $\quad a = \frac{dv}{dt} = \frac{d}{dt}(v_0 - kx) = -kv$

or $\quad a = -k(v_0 - kx)$

At $t = 0$: $\quad a = 2700 \frac{1}{\text{s}}(900 \text{ ft/s} - 0)$

or $\quad a_0 = -2.43 \times 10^6 \text{ ft/s}^2$ ◀

(*b*) We have $\quad \frac{dx}{dt} = v = v_0 - kx$

At $t = 0, x = 0$: $\quad \int_0^x \frac{dx}{v_0 - kx} = \int_0^t dt$

or $\quad -\frac{1}{k}[\ln(v_0 - kx)]_0^x = t$

or $\quad t = \frac{1}{k} \ln\left(\frac{v_0}{v_0 - kx}\right) = \frac{1}{k} \ln\left(\frac{1}{1 - \frac{k}{v_0}x}\right)$

When $x = 3.9$ in.: $\quad t = \frac{1}{2700 \frac{1}{\text{s}}} \ln\left[\frac{1}{1 - \frac{2700 \text{ 1/s}}{900 \text{ ft/s}}\left(\frac{3.9}{2} \text{ ft}\right)}\right]$

or $\quad t = 1.366 \times 10^{-3} \text{ s}$ ◀

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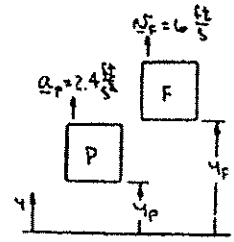
PROBLEM 11.185

A freight elevator moving upward with a constant velocity of 6 ft/s passes a passenger elevator which is stopped. Four seconds later, the passenger elevator starts upward with a constant acceleration of 2.4 ft/s^2. Determine (*a*) when and where the elevators will be at the same height, (*b*) the speed of the passenger elevator at that time.

SOLUTION

(*a*) For $t \geq 0$: $y_F = 0 + v_F t$

$t \geq 4$ s: $y_P = 0 + 0(t-4) + \frac{1}{2} a_P (t-4)^2$



When $y_F = y_P$

$$(6 \text{ ft/s})t = \frac{1}{2}(2.4 \text{ ft/s}^2)(t-4)^2$$

Expanding and simplifying

$$t^2 - 13t + 16 = 0$$

Solving $t = 1.3765$ s and $t = 11.6235$ s

Most require $t > 4$ s $t = 11.62$ s ◀

At $t = 11.6235$ s: $y_F = (6 \text{ ft/s})(11.6235 \text{ s})$

or $y_F = y_P = 69.7$ ft ◀

(*b*) For $t \geq 4$ s: $v_P = 0 + a_P(t-4)$

At $t = 11.6235$ s: $v_P = (2.4 \text{ ft/s}^2)(11.6235 - 4) \text{ s}$

or $v_P = 18.30$ ft/s ◀

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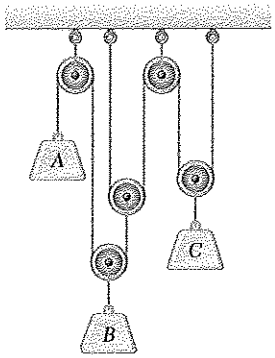
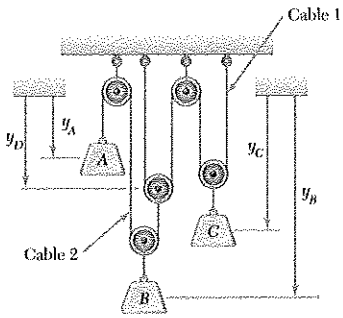


PROBLEM 11.186

Block C starts from rest at $t = 0$ and moves upward with a constant acceleration of 25 mm/s^2. Knowing that block A moves downward with a constant velocity of 75 mm/s, determine (a) the time for which the velocity of block B is zero, (b) the corresponding position of block B.

SOLUTION

The cable lengths are constant.



$$L_1 = 2y_C + 2y_D + \text{constant}$$
$$L_2 = y_A + y_B + (y_B - y_D) + \text{constant}$$

Eliminate y_D.

$$L_1 + 2L_2 = 2y_C + 2y_D + 2y_A + 2y_B + 2(y_B - y_D) + \text{constant}$$
$$2(y_C + y_A + 2y_B) = \text{constant}$$

Differentiate to obtain relationships for velocities and accelerations, positive downward.

$$v_C + v_A + 2v_B = 0 \tag{1}$$

$$a_C + a_A + 2a_B = 0 \tag{2}$$

Initial velocities. $(v_C)_0 = 0, \quad (v_A)_0 = 75$ mm/s

From (1), $(v_B)_0 = -\frac{1}{2}[(v_C)_0 + (v_A)_0] = -37.5$ mm/s

Accelerations. $a_C = -25$ mm/s $\quad a_A = 0$

From (2), $a_B = -\frac{1}{2}(a_C + a_A) = 12.5$ mm/s

Motion of block B.

$$v_B = (v_B)_0 + a_B t = -37.5 + 12.5t$$
$$\Delta y_B = (v_B)_0 t + \frac{1}{2} a_B t^2 = -37.5t + 6.25t^2$$

(a) Time at $v_B = 0$. $\quad -37.5 + 12.5t = 0 \quad t = 3.00$ s ◀

(b) Corresponding position. $\quad \Delta y_B = (-37.5)(3.00) + (6.25)(3.00)^2 \quad \Delta y_B = -56.25$ mm ◀

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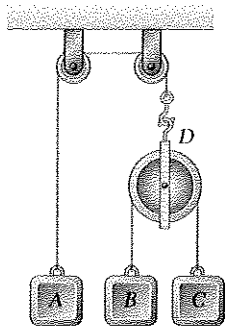
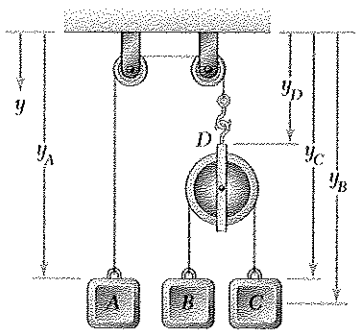


PROBLEM 11.187

The three blocks shown move with constant velocities. Find the velocity of each block, knowing that the relative velocity of A with respect to C is 300 mm/s upward and that the relative velocity of B with respect to A is 200 mm/s downward.

SOLUTION



From the diagram

Cable 1: $y_A + y_D = \text{constant}$

Then $v_A + v_D = 0$ (1)

Cable 2: $(y_B - y_D) + (y_C - y_D) = \text{constant}$

Then $v_B + v_C - 2v_D = 0$ (2)

Combining Eqs. (1) and (2) to eliminate v_D

$$2v_A + v_B + v_C = 0 \quad (3)$$

Now $v_{A/C} = v_A - v_C = -300 \text{ mm/s}$ (4)

and $v_{B/A} = v_B - v_A = 200 \text{ mm/s}$ (5)

Then $(3) + (4) - (5) \Rightarrow$

$$(2v_A + v_B + v_C) + (v_A - v_C) - (v_B - v_A) = (-300) - (200)$$

or $\mathbf{v}_A = 125 \text{ mm/s} \uparrow$ ◀

and using Eq. (5) $v_B - (-125) = 200$

or $\mathbf{v}_B = 75 \text{ mm/s} \downarrow$ ◀

Eq. (4) $-125 - v_C = -300$

or $\mathbf{v}_C = 175 \text{ mm/s} \downarrow$ ◀

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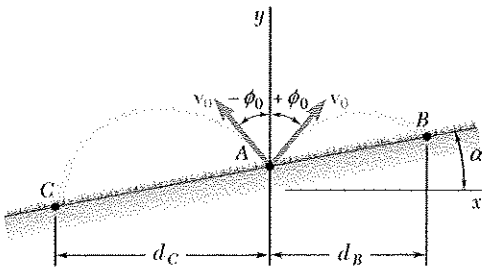


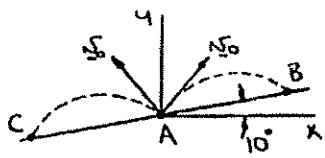
PROBLEM 11.188

An oscillating water sprinkler at Point A rests on an incline which forms an angle α with the horizontal. The sprinkler discharges water with an initial velocity $\mathbf{v}_0$ at an angle ϕ with the vertical which varies from $-\phi_0$ to $+\phi_0$. Knowing that $v_0 = 30$ ft/s, $\phi_0 = 40°$, and $\alpha = 10°$, determine the horizontal distance between the sprinkler and Points B and C which define the watered area.

SOLUTION

First note

$$(v_0)_x = v_0 \sin\phi = (30 \text{ ft/s})\sin\phi$$
$$(v_0)_y = v_0 \cos\phi = (30 \text{ ft/s})\cos\phi$$



Also, along incline Cab

$$y = x\tan 10°$$

Horizontal motion. (Uniform)

$$x = 0 + (v_0)_x t = (30\sin\phi)t \quad \text{or} \quad t = \frac{x}{30\sin\phi}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_0)_y t - \frac{1}{2}gt^2 = (30\cos\phi)t - \frac{1}{2}gt^2$$

Substituting for t

$$y = (30\cos\phi)\left(\frac{x}{30\sin\phi}\right) - \frac{1}{2}g\left(\frac{x}{30\sin\phi}\right)^2$$

$$= \frac{x}{\tan\phi} - \frac{g}{1800}\frac{x^2}{\sin^2\theta} \qquad (g = 32.2 \text{ ft/s}^2)$$

At B: $\phi = 40°$, $x = d_B$: $\quad d_B\tan 10° = \dfrac{d_B}{\tan 40°} - \dfrac{32.2}{1800}\dfrac{d_B^2}{\sin^2 40°}$

or $\qquad d_B = 23.5 \text{ ft}$ ◀

At C: $\phi = -40°$, $x = -d_C$: $\quad -d_C\tan 10° = \dfrac{-d_C}{\tan(-40°)} - \dfrac{32.2}{1800}\dfrac{(-d_C)^2}{\sin^2(-40°)}$

or $\qquad d_C = 31.6 \text{ ft}$ ◀

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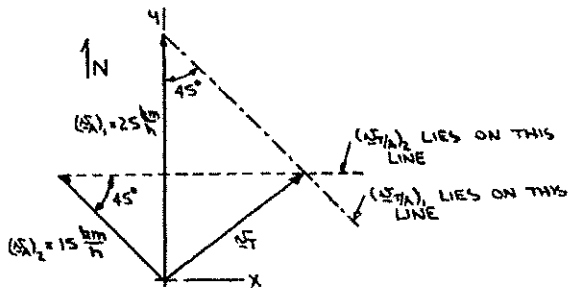
PROBLEM 11.189

As the driver of an automobile travels north at 25 km/h in a parking lot, he observes a truck approaching from the northwest. After he reduces his speed to 15 km/h and turns so that he is traveling in a northwest direction, the truck appears to be approaching from the west. Assuming that the velocity of the truck is constant during the period of observation, determine the magnitude and the direction of the velocity of the truck.

SOLUTION

We have $$\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$$

Using this equation, the two cases are then graphically represented as shown.



From the diagram

$$(v_T)_x = 25 - 15\sin 45°$$
$$= 14.3934 \text{ km/h}$$
$$(v_T)_y = 15\sin 45°$$
$$= 10.6066 \text{ km/h}$$

$$\mathbf{v}_T = 17.88 \text{ km/h} \measuredangle 36.4° \blacktriangleleft$$

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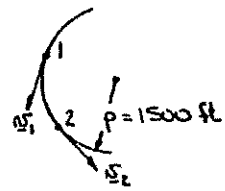
PROBLEM 11.190

The driver of an automobile decreases her speed at a constant rate from 45 to 30 mi/h over a distance of 750 ft along a curve of 1500-ft radius. Determine the magnitude of the total acceleration of the automobile after the automobile has traveled 500 ft along the curve.

SOLUTION

First note $v_1 = 45 \text{ mi/h} = 66 \text{ ft/s}$

$v_2 = 30 \text{ mi/h} = 44 \text{ ft/s}$



We have uniformly decelerated motion

$$v^2 = v_1^2 + 2a_t(s - s_1)$$

When $v = v_2$: $(44 \text{ ft/s})^2 = (66 \text{ ft/s})^2 + 2a_t(750 \text{ ft})$

or $a_t = -1.61333 \text{ ft/s}^2$

Then when $\Delta s = 500$ ft: $v^2 = (66 \text{ ft/s})^2 + 2(-1.61333 \text{ ft/s}^2)(500 \text{ ft})$

$= 2742.67 \text{ ft}^2/\text{s}^2$

Now $a_n = \dfrac{v^2}{\rho}$

$= \dfrac{2742.67 \text{ ft}^2/\text{s}^2}{1500 \text{ ft}}$

$= 1.82845 \text{ ft/s}^2$

Finally $a^2 = a_t^2 + a_n^2$

$= (-1.61333 \text{ ft/s}^2)^2 + (1.82845 \text{ ft/s}^2)^2$

or $a = 2.44 \text{ ft/s}^2$ ◀

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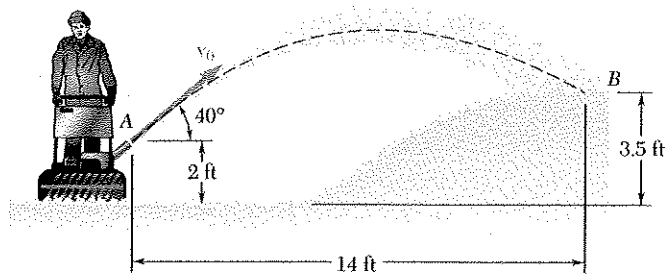
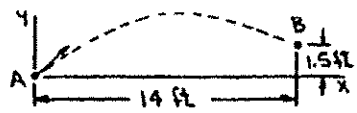


PROBLEM 11.191

A homeowner uses a snowblower to clear his driveway. Knowing that the snow is discharged at an average angle of 40° with the horizontal, determine the initial velocity v_0 of the snow.

SOLUTION



First note $(v_x)_0 = v_0 \cos 40°$

$(v_y)_0 = v_0 \sin 40°$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At B: $$14 = (v_0 \cos 40°)t \quad \text{or} \quad t_B = \frac{14}{v_0 \cos 40°}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2 \qquad (g = 32.2 \text{ ft/s}^2)$$

At B: $$1.5 = (v_0 \sin 40°)t_B - \frac{1}{2} g t_B^2$$

Substituting for t_B

$$1.5 = (v_0 \sin 40°)\left(\frac{14}{v_0 \cos 40°}\right) - \frac{1}{2} g \left(\frac{14}{v_0 \cos 40°}\right)^2$$

or $$v_0^2 = \frac{\frac{1}{2}(32.2)(196)/\cos^2 40°}{-1.5 + 14 \tan 40°}$$

or $$v_0 = 22.9 \text{ ft/s} \blacktriangleleft$$

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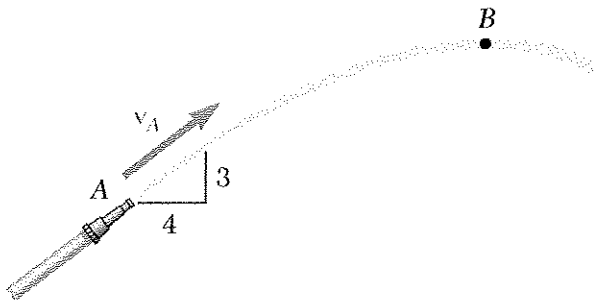


PROBLEM 11.192

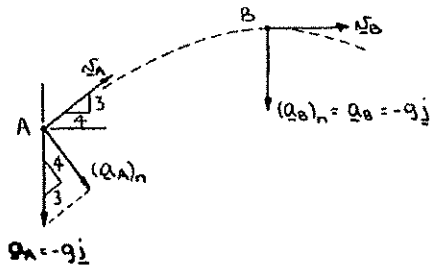
From measurements of a photograph, it has been found that as the stream of water shown left the nozzle at A, it had a radius of curvature of 25 m. Determine (a) the initial velocity $\mathbf{v}_A$ of the stream, (b) the radius of curvature of the stream as it reaches its maximum height at B.

SOLUTION

(a) We have $(a_A)_n = \dfrac{v_A^2}{\rho_A}$

or $v_A^2 = \left[\dfrac{4}{5}(9.81 \text{ m/s}^2)\right](25 \text{ m})$

or $v_A = 14.0071 \text{ m/s}$



$\mathbf{v}_A = 14.01 \text{ m/s} \angle 36.9°$ ◀

(b) We have $(a_B)_n = \dfrac{v_B^2}{\rho_B}$

Where $v_B = (v_A)_x = \dfrac{4}{5} v_A$

Then $\rho_B = \dfrac{\left(\frac{4}{5} \times 14.0071 \text{ m/s}\right)^2}{9.81 \text{ m/s}^2}$

or $\rho_B = 12.80 \text{ m}$ ◀

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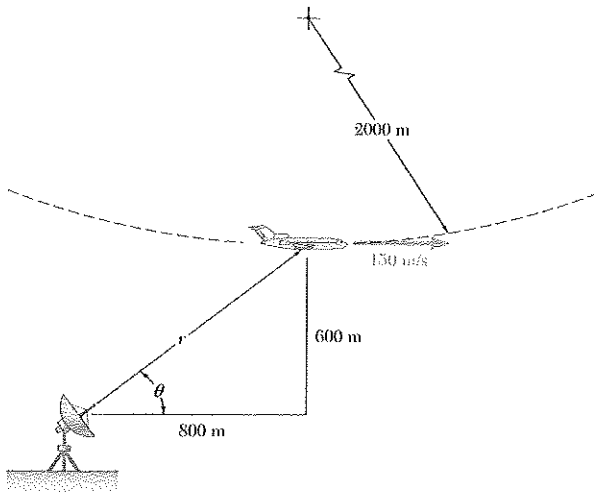


PROBLEM 11.193

At the bottom of a loop in the vertical plane, an airplane has a horizontal velocity of 150 m/s and is speeding up at a rate of 25 m/s^2. The radius of curvature of the loop is 2000 m. The plane is being tracked by radar at O. What are the recorded values of $\dot{r}$, $\ddot{r}$, $\dot{\theta}$ and $\ddot{\theta}$ for this instant?

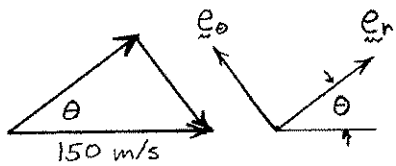
SOLUTION

Geometry. The polar coordinates are

$$r = \sqrt{(800)^2 + (600)^2} = 1000 \text{ m} \quad \theta = \tan^{-1}\left(\frac{600}{800}\right) = 36.87°$$

Velocity Analysis.

$$\mathbf{v} = 150 \text{ m/s} \rightarrow$$



$$v_r = 150\cos\theta = 120 \text{ m/s}$$

$$v_\theta = -150\sin\theta = -90 \text{ m/s}$$

$$v_r = \dot{r} \qquad \dot{r} = 120 \text{ m/s} \blacktriangleleft$$

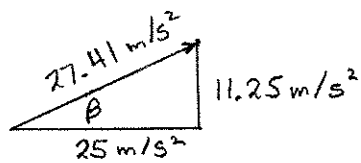
$$v_\theta = r\dot{\theta} \quad \dot{\theta} = \frac{v_\theta}{r} = -\frac{90}{1000}$$

$$\dot{\theta} = -0.0900 \text{ rad/s} \blacktriangleleft$$

Acceleration analysis.

$$a_t = 25 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(150)^2}{2000} = 11.25 \text{ m/s}^2$$



$$\mathbf{a} = 25 \text{ m/s}^2 \rightarrow + 11.25 \text{ m/s}^2 \uparrow = 27.41 \text{ m/s}^2 \measuredangle 24.23°$$

$$\beta = 24.23°$$

$$\theta - \beta = 12.64°$$

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PROBLEM 11.193 (Continued)

$$a_r = a\cos(\theta - \beta) = 27.41\cos 12.64° = 26.74 \text{ m/s}^2$$

$$a_\theta = -a\sin(\theta - \beta) = -27.41\sin 12.64° = -6.00 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \qquad \ddot{r} = a_r + r\dot{\theta}^2$$

$$\ddot{r} = 26.74 + (1000)(0.0900)^2 \qquad \ddot{r} = 34.8 \text{ m/s}^2 \blacktriangleleft$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\ddot{\theta} = \frac{a_\theta}{r} - \frac{2\dot{r}\dot{\theta}}{r}$$

$$= \frac{-6.00}{1000} - \frac{(2)(120)(-0.0900)}{1000} \qquad \ddot{\theta} = -0.0156 \text{ rad/s}^2 \blacktriangleleft$$

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(*b*) Based on Eq. (1) and the results of Part *a*, it can be concluded that d increases in value as α increases in value from 0 to 45° and then d decreases as α is further increased. Thus, $d_{\max}$ occurs for the value of α closest to 45° and for which the water just passes over the first row of corn plants. At this row, $x_{\text{com}} = 1.5$ m

so that
$$t_{\text{com}} = \frac{1.5}{8\cos\alpha}$$

Also, with $y_{\text{com}} = h$, we have

$$h = (8\sin\alpha)t_{\text{com}} - \frac{1}{2}gt_{\text{com}}^2$$

Substituting for t_{com} and noting $h = 1.8$ m,

$$1.8 = (8\sin\alpha)\left(\frac{1.5}{8\cos\alpha}\right) - \frac{1}{2}g\left(\frac{1.5}{8\cos\alpha}\right)^2$$

or
$$1.8 = 1.5\tan\alpha - \frac{2.25g}{128\cos^2\alpha}$$

Now
$$\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$$

Then
$$1.8 = 1.5\tan\alpha - \frac{2.25(9.81)}{128}(1 + \tan^2\alpha)$$

or
$$0.172441\tan^2\alpha - 1.5\tan\alpha + 1.972441 = 0$$

Solving
$$\alpha = 58.229° \quad \text{and} \quad \alpha = 81.965°$$

From the above discussion, it follows that $d = d_{\max}$ when

$$\alpha = 58.2° \blacktriangleleft$$

Finally, using Eq. (1)

$$d = \frac{64}{9.81}\sin(2\times 58.229°)$$

or
$$d_{\max} = 5.84 \text{ m} \blacktriangleleft$$

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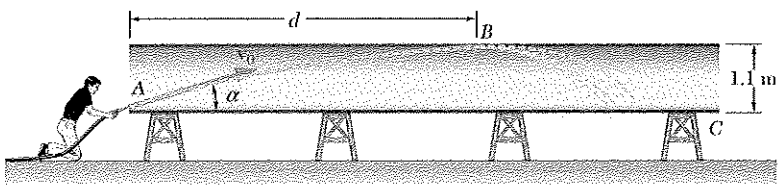
PROBLEM 11.116



A worker uses high-pressure water to clean the inside of a long drainpipe. If the water is discharged with an initial velocity $\mathbf{v}_0$ of 11.5 m/s, determine (*a*) the distance *d* to the farthest Point *B* on the top of the pipe that the worker can wash from his position at *A*, (*b*) the corresponding angle α.

SOLUTION

First note $(v_x)_0 = v_0 \cos\alpha = (11.5 \text{ m/s}) \cos\alpha$

$(v_y)_0 = v_0 \sin\alpha = (11.5 \text{ m/s}) \sin\alpha$

By observation, $d_{\max}$ occurs when $y_{\max} = 1.1$ m.

Vertical motion. (Uniformly accelerated motion)

$$v_y = (v_y)_0 - gt \qquad y = 0 + (v_y)_0 t - \frac{1}{2}gt^2$$

$$= (11.5 \sin\alpha) - gt \qquad = (11.5 \sin\alpha)t - \frac{1}{2}gt^2$$

When $y = y_{\max}$ at B, $(v_y)_B = 0$

Then $(v_y)_B = 0 = (11.5 \sin\alpha) - gt$

or $$t_B = \frac{11.5 \sin\alpha}{g} \quad (g = 9.81 \text{ m/s}^2)$$

and $$y_B = (11.5 \sin\alpha)t_B - \frac{1}{2}gt_B^2$$

Substituting for t_B and noting $y_B = 1.1$ m

$$1.1 = (11.5 \sin\alpha)\left(\frac{11.5 \sin\alpha}{g}\right) - \frac{1}{2}g\left(\frac{11.5 \sin\alpha}{g}\right)^2$$

$$= \frac{1}{2g}(11.5)^2 \sin^2\alpha$$

or $$\sin^2\alpha = \frac{2.2 \times 9.81}{11.5^2} \qquad \alpha = 23.8265°$$

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